# HAPPINESS AND ALTRUISM: A DYNAMIC MODEL OF VIRTUOUS BEHAVIOUR

By

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**Abstract**

This paper aims at formulating a model of altruism, virtuous capital and happiness. We want to show the implications of altruism on individual behaviour. Our interest lies in those consumption choices of material goods, monetary donations and altruistic behaviour that lead individuals to reach the maximum level of happiness. By using the concepts of consumption capital and addictional goods, we model virtuous behaviour and introduce it as an argument of the utility function. After solving the dynamic optimization problem we find that the growth of material goods consumption responds positively to the rate of return on wealth. Complementarity between $x$ and $V$ positively affects the rate of growth of $x$. The growth of monetary donations responds positively to the rate of return of material wealth and to the complementarity between $V$ and $d$. We empirically check the relation between the consumption of material goods and donations and their respective determinants and other socio-economic variables, using data collected through a questionnaire. We find that the return on wealth positively affects both material goods consumption and monetary donations. Finally, we obtain that in the median term people engaged in volunteering activities have a consumption higher by 20,8% with respect to individuals not involved in those kinds of activities, while in the median term people engaged in volunteering activities make donations higher by 42,47% with respect to individuals not involved in those activities.

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## Introduction

With the reformulation of consumer theory realized by Becker (1964, 1965) and Michael and Becker (1973), the *household production function* concept was introduced into economic theory. By the employ of this function households become active maximizers of utility, because an individual's objects of choice are no longer goods bought on markets, but *commodities* that they produce by use of their time, skills, training, human capital and other inputs. This new approach make it possible to explain a wide range of important phenomena not clarified by economic theory up to that time, and to introduce two other important concepts: *consumption capital* and *addictional goods*. These ideas have been formalized by authors such as Ryder and Heal (1973), Stigler and Becker (1977), Boyer (1978), Iannacone (1986), Becker and Murphy (1988).

Consumption capital is represented, as defined by Iannacone (1986), by the skills and experience of individuals that enable them to produce satisfaction. This capital is a fundamental determinant of one's ability to produce and appreciate goods. Addictional goods are those kinds of goods which generate a self-feeding loop, namely, the satisfaction individuals obtain from such fruition over a period of time is correlated with the quantity consumed in all previous periods.

Both these concepts have been employed to study cultural goods consumption (Scandizzo 1992, 1993) and religious behaviour (Iannacone 1986). We apply them to the case of altruistic behaviour. In particular, we formulate a model in which utility function arguments are represented by material goods consumption, altruism and virtuous capital. Our aim is to show the implications of altruism and accumulation of virtuous capital on individual behaviour. We are interested in those consumption choices of material goods, monetary donations and altruistic behaviour that lead individuals to reach the maximum level of happiness.

Our utility measures are represented by the individuals' level of happiness at a certain moment of time. By virtuous behaviour we mean an altruistic conduct that manifests itself by monetary donations and by involvement in volunteering organizations. Consumption capital is represented by the experience, skills and level of consciousness individuals reach thanks to altruistic behaviour, as set out in the works of Iannacone (1986) and Scandizzo (1992, 1993). Finally, we consider donations as addictional goods: past donations influence the satisfaction individuals obtain from what is given today, thanks to the virtuous capital accumulated.

Our reference model is that of Scandizzo (1993), which analyses the implications of cultural consumption for individual behaviour and economic growth, as anticipated above. While Scandizzo introduces cultural goods consumption as a control variable, we focus our attention on monetary donations and virtuous capital. In particular, donations determine a double effect on utility function. On the one hand, they directly influence individual's utility because they appear as an explanatory variable in the utility function; on the other hand, donations influence indirectly an individual's utility by raising the stock of virtuous capital. Moreover, the higher the stock of virtuous capital, the greater the satisfaction individuals will obtain from future donations. Consumption of material goods also raises an individual's utility. However a trade-off exists between donations and material goods consumption, given the wealth constraint. A higher stock of virtuous capital also makes it possible to obtain more pleasure from material goods consumption.

The model did not appear to be well-behaved. The growth of material goods consumption responds positively to the rate of return on wealth and the complementarity between $x$ and $V$ positively affects the rate of growth of $x$. The growth of monetary donations responds positively to the rate of return of material wealth and to the complementarity between $V$ and $d$.

The remainder of the work is organized as follows. Section 1 reviews the literature on addictional goods and consumption capital. Section 2 describes the model. Section 3 introduces the database. Section 4 develops an econometric analysis. Finally, Section 5 summarizes the results.

## 1. Review of literature on consumption capital and addictional goods

The concepts of *addictional goods* and *consumption capital* have been the subject of several studies in economic literature. They have been formalized by some authors such as Ryder and Heal (1973), Stigler and Becker (1977), Boyer (1978), Iannacone (1986), Becker and Murphy (1988). Their introduction in the economic literature has been possible thanks to the reformulation of consumer theory realized by Becker[1] (1964, 1965) and Michael and Becker (1973), which generates many new implications about behaviours that are consistent with stable tastes.

As is well-known, the traditional theory of consumer behaviour rests on the view that the consumer tries to maximize utility, U, which is obtained directly from the goods $x_i$ purchased in the marketplace

$$U = u(x_1, x_2 \ldots\ldots x_n) \tag{1}$$

subject to a constraint represented by money income

$$I = \sum_{i=1}^{n} x_i p_i \tag{2}$$

According to this theory, variations in demands which are not related to changes in real income and relative prices are attributed to changes in tastes. Together these three factors, income, prices and tastes, fully explain consumption behaviour. The single important behavioural law which emerges from this approach is that income-compensated changes in the relative price of any goods lead to changes in the opposite direction in the quantity demanded of those goods[2].

The new approach transforms the family from a passive maximizer of the utility derived from market purchases into an active maximizer engaged in production and investment activities. The utility function is now changed because the arguments that households choose are no longer goods and services bought in the marketplace. They maximize a utility function of different objects of choice, called commodities, that they produce using their own time, skills, training, human capital and other inputs. Unlike the products of a commercial firm, household commodities are consumed by family members rather than being sold.

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[1] One important statement of Becker (1976) summarizes the new approach well. He asserts that "…all human behaviour can be viewed as involving participants who maximize their utility from a stable set of preferences and accumulate an optimal amount of information and other inputs in a variety of markets…".

2 Michael and Becker (1973) point out some fundamental weaknesses of this approach. The first weak point is represented by the reliance on tastes when income and prices are not able to explain human behaviour. The problem arises because economists can neither explain how tastes are formed nor predict their effects. Furthermore, assuming that utility is derived from goods and services purchased in the marketplace, the traditional theory has generally been formulated in terms of monetary prices and monetary income. Hence, its application has tended to be restricted to the market sector where transactions are easily quantified by money. Many other decisions about the allocation of consumer non-market time and decisions about choice of religion, marriage mate, family size, divorce, political party, have been ignored by economists. With their reformulation, Michael and Becker (1973) are able to explain a wide range of important phenomena with which the traditional theory does not cope.

Formally, the household's utility function would be

$$U = u(Z_1, Z_2 .....Z_n) \tag{3}$$

with

$$Z_i = f_i(X_{1i}, .....X_{ki}, t_{1i} .....t_{li}, S_i .....S_l, Y_i) \qquad i=1 \ldots\ldots m \tag{4}$$

where $Z_i$ are the commodities entering the utility function, $f_i$ is the production function for the *ith* , $X_{ji}$ is the quantity of the *jth* market goods or service used in the production of the *ith* commodity, $t_{ij}$ is the *jth* person's own time input, $S_j$ the *jth* person's human capital, and $Y_i$ represents all other inputs. The utility function is maximized subject to the production function constraints (4) and a constraint on the household's available time

$$T = t_w + \sum_{i=1}^{n} t_i \tag{5}$$

and the usual income constraint

$$I = \sum_{i=1}^{n} p_i X_i \tag{6}$$

where $t_w$ and $t_i$ are the household's time spent in the labour market and in producing $Z_i$ respectively, and $p_i$ and $X_i$ are the price and quantity of the market good input used in producing $Z_i$. By collapsing time and income constraints into a single resources constraint, it is possible to maximize the utility function subject to the constraints of the production function (4) and the full resources constraint.

The first order conditions for maximization with respect to the commodities imply

$$\frac{MU_i}{MU_j} = \frac{[(w * \frac{dt_i}{dZ_i}) + (p_i * \frac{dx_i}{dZ_i})]}{[(w * \frac{dt_j}{dZ_j}) + (p_j * \frac{dx_j}{dZ_j})]} \equiv \frac{\pi_i}{\pi_j} \tag{7}$$

The ratio of the marginal utilities of any two commodities $Z_i$ and $Z_j$, $MU_i/MU_j$, must equal the ratio of their marginal costs, $\pi_i / \pi_j$, where the derivatives are marginal input-output coefficients. These marginal costs are the shadow prices of the $Z_i$ that are determined by the prices of market goods and time, and by the productivity of each in producing $Z_i$.

It is also possible to differentiate the utility function with respect to all production factors to determine their optimal use

$$\frac{\partial U / \partial Z_i * \partial Z_i / \partial f_{ik}}{\partial U / \partial Z_j * \partial Z_j / \partial f_{jl}} \equiv \frac{MU_i}{MU_j} * \frac{MP_{ik}}{MP_{jl}} = \frac{p_{fik}}{p_{fil}} \tag{8}$$

where $f_{ik}$ is the factor k (either goods or time) used in producing $Z_i$ and $f_{jl}$ is the factor l (either goods or time) used in producing $Z_j$. When both factors are used in the same production function

(i=j), the condition reduces to an equality of the ratio of marginal products to the ratio of the factor prices.

Now households respond to changes in prices and productivities of factors, to changes in the relative shadow prices of commodities and to changes in their full real income, as they attempt to minimize their costs of production and to maximize their utility. A reduction in the prices of some production factors will shift the production process towards techniques that are more intensive in the use of that factor and towards commodities that use the factor relatively intensely.

The new approach is not in conflict with the traditional implications regarding household responses to changes in relative prices or real income. On the contrary, an important advantage is its great emphasis on income and price effects and, correspondingly, its reduced emphasis on the role of tastes in interpreting behaviour[3].

Finally, Becker and Murphy (1988) point out that many phenomena previously thought to be irrational are consistent with optimization according to stable preferences. An example is represented by consumption of additional goods. The two authors claim that "…addictions, even strong ones, are usually rational in the sense of involving forward-looking maximization with stable preferences. Our claim is even stronger: a rational framework permits new insights into addictive behaviour…." (Becker and Murphy (1988)). A final result the authors reach is that a permanent change in the price of addictive goods may have a small initial effect on demand, but the effect grows in the long run. In particular, the greater is the level of addiction, the higher the long run effect on consumption of a change in price.

### 1.1 The formalization of addictional behaviour

As noted before, a rational framework allows us to gain new insights into addictive behaviour, introducing the key concepts of addictional goods and consumption capital. Scandizzo (1992) defines as "…addictional goods [those which] generate a self-feeding fruition loop, which means that the satisfaction consumers obtain from such fruition in a period of time is positively correlated with the quantity consumed in all previous periods…".

Ryder and Heal (1973) are the first to formalize this kind of behaviour. They introduce into the utility function a new variable, z, which can be interpreted either as the customary level of consumption, or as the expected level of consumption. Instantaneous satisfaction then depends both on instantaneous consumption and on the expected consumption level. The justification for including such a variable is obvious: it is that the amount of satisfaction that a man derives from consuming a given bundle of goods depends not only on that bundle, but also on his past consumption and on his general social environment.

In particular, the variable z(t) can be formalized in the following way

$$z(t) = \rho e^{-\rho t} \int_{-\infty}^{t} e^{\rho t} c(\tau) d\tau \qquad (9)$$

[3] Using this reformulation, Stigler and Becker (1977) survey four classes of phenomena considered to be inconsistent with the stability of tastes: addiction, habitual behaviour, advertising and fashions. In each case the authors offer an alternative explanation that not only reconcile these phenomena with stability of tastes, but also try to show that the hypothesis of stable tastes yields more useful predictions about observable behaviour.

where ρ>0 and c(τ) is the average level of per capita consumption in the community at time τ. z(t) is thus a weighted average of past consumption levels, with the weights declining exponentially into the past. The larger is ρ, the less weight is given to past consumption in determining z(t). It is the variable z(t) that is regarded as the expected consumption level at time t.

Ryder and Heal solve a problem of optimal control, where they wish to choose a consumption path c(t), for t≥0 that will maximize

$$J[c(\cdot)] = \int_0^{\infty} e^{-\delta t} u[c(t), z(t)]dt \tag{10}$$

subject to

$$\overset{\circ}{k} = f(k) - \lambda k - c \tag{11}$$
$$0 \le c \le f(k) \tag{12}$$
$$\overset{\circ}{Z} = \rho(c - z) \tag{13}$$
$$Z(0) = z_0 > 0, k(0) = k_0 > 0 \tag{14}$$

where equation (11) represents the law of motion of capital-labour ratio, in which λ>0 is the sum of the growth rate of the labour force and the depreciation rate of capital, f is the production function, c is consumption per worker. Differentiating equation (9) with respect to z, the authors obtain a differential equation for z (13). Finally, at the planning date t=0 they are faced with historically given endowment of capital $k_0$ and past consumption $z_0$. In order to solve this problem, to discuss marginal utilities and marginal rates of substitution between consumption at various dates, they use the concept of derivative of a functional, as worked out by Volterra (1959)[4].

Two authors who rely on the concepts worked out by Volterra are Becker and Murphy (1988). According to them, goods are addictive if and only if consumptions of the goods at different moments in time are complements. Moreover, the degree of addiction is stronger when the complementarity in consumption is greater. The link between addiction and complementarity

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[4] The Volterra derivative of J[c(·)] is itself a functional which may be differentiated in the same way to obtain second and higher order Volterra derivatives, J''[c(·);$t_1$, $t_2$]. When welfare is a functional of a continuous stream of consumption, the marginal utility of consumption at date $t_1$ is the Volterra derivative J'[c(·);$t_1$]. The marginal rate of substitution between consumption at dates $t_1$ and $t_2$ is the ratio of marginal utilities

$$R[c(\cdot), t_1, t_2] = \frac{J'[c(\cdot); t_1]}{J'[c(\cdot); t_2]} \tag{15}$$

If there is a small increment in consumption at date $t_3$, the effect on the marginal rate of substitution between $t_1$ and $t_2$ is another Volterra derivative:

$$R'[c(\cdot), t_1, t_2, t_3] = \frac{\{J'[c(\cdot); t_2]J''[c(\cdot); t_1, t_3] - \} J'[c(\cdot); t_1]J''[c(\cdot); t_1, t_3]}{\{J'[c(\cdot); t_2]\}^2} \tag{16}$$

If R'[c(·),$t_1$, $t_2$ ; $t_3$]>0, a small increment at $t_3$ shifts preferences from $t_2$ to $t_1$. In this case there will be complementarity between $t_3$ and $t_1$.This represents the so-called *distant complementarity*. If R'<0, the increment at $t_3$ shifts preferences from $t_1$ to $t_2$, determining the so-called *adjacent complementarity* between $t_2$ and $t_3$. If R'=0 an increment in $t_3$ does not affect preferences between $t_1$ and $t_2$. In this case the preference functional is independent.

implies that an anticipated increase in future prices of addictive goods lowers current consumption. The longer future price changes are anticipated, the greater is their effect on the current consumption of addictive goods. The reason is that the longer a future price rise is anticipated, the greater is the reduction in past consumption of the goods. Therefore, the stock of capital carried in to the present period would be smaller.

They set out first order conditions for utility maximization and consider dynamic aspects of addictive consumption. In particular they assume that utility of an individual at any moment depends on the consumption of two goods, c and y. Utility depends also on past consumption, which determines a process of learning by doing, as summarized by the stock of consumption capital S. In terms of equations the problem can be stated as follow

$$MaxU(0) = \int_0^{\infty} e^{-\sigma t} u[y(t), c(t), S(t)]dt \tag{17}$$

subject to the budget constraint

$$\int_0^T e^{-rt}[y(t) + p_c(t)c(t) + p_d(t)D(t)] \leq A_0 + \int_0^T e^{-rt} w(S(t))dt \tag{18}$$

and to the investment equation

$$\overset{\circ}{S}(t) = c(t) - \delta S(t) - h[D(t)] \tag{19}$$

where $A_0$ represents the initial value of assets, r is the rate of interest constant over time, w(S) are the earnings, δ measures the exogenous rate of depreciation of physical and mental capital, and D(t) represents expenditures on endogenous depreciation or appreciation.

After having found the optimal paths of y(t) and c(t) using the first order conditions, Becker and Murphy (1988) analyse the dynamic behaviour of c and S near a steady state. They show that steady state consumption of addictive goods is unstable when the degree of addiction is strong. Consumption rises over time when above unstable steady state levels, and it falls over time when below unstable steady states. The authors underline how "…Unstable steady states prove to be an important tool in the analysis of addictive behaviour. They allow us to explain rational "pathological" addictions, in which a person's consumption of goods continues to increase over time even though he fully anticipates the future negative [effects]…….. They are also important in explaining *normal* addictions that may involve rapid increases in consumption only for a while"[5].

Two other economists who formalize the concepts of addictional consumption are Stigler and Becker (1977). They assume that individuals increase their desire for these goods over time, which in utility language means they have an increasing marginal utility. However their explanation is not changing tastes[6], but the accumulation of *consumption capital* by the consumer,

[5] Becker, Grossman and Murphy (1994) empirically test the model of Becker and Murphy (1988). They check the model of rational addiction by considering the response of cigarette consumption to a change in cigarette prices. The authors examine whether lower past and future prices for cigarettes raise current cigarette consumption. The empirical results tend to support the implication of addictive behaviour that long-run responses exceed short-run responses. Another empirical work which reviews many estimates of the demand for cigarettes is that of Mullahy (1985)

[6] In his numerous works, Becker underlines the importance for his results of the hypothesis of stable preferences. In particular, in his book *The Economic Approach to Human Behaviour* (1976) he underlies how "…all human behaviours

distinguishing between beneficial addiction and harmful addiction. In analysing beneficial addiction, they use an unchanging utility function that depends on two produced commodities

$$U = U(M, Z) \tag{20}$$

where M measures the amount of music appreciation produced and consumed, and Z production and consumption of other commodities. M is created by a function that depends on the time allocated to music, $t_m$, and the training and other human capital conducive to music appreciation, $S_m$

$$M = M_m(t_m, S_m) \tag{21}$$

An increase in this music capital increases the productivity of time spent listening to or devoted in other ways to music. The amount of appreciation produced at any moment j, $M_j$, would depend on the time allocated to music and the human music capital at j: $t_{mj}$ and $S_{mj}$ respectively. The latter in turn is produced partly through "learning by doing" by accumulating the effects of earlier music appreciation. By definition the addiction is beneficial if

$$\frac{\partial S_{mj}}{\partial M_{j-v}} > 0 \text{ for all } v \tag{22}$$

The optimal allocation between M and Z is determined by the equality between the ratio of their marginal utilities and the ratio of their shadow prices

$$\frac{MU_{mj}}{MU_{zj}} = \frac{\frac{\partial U}{\partial M_j}}{\frac{\partial U}{\partial Z_j}} \equiv \frac{\pi_{mj}}{\pi_{zj}} \tag{23}$$

The shadow price equals the marginal cost of adding a unit of commodity output. The marginal cost of music appreciation M is now reduced by the positive effect on subsequent music human capital of the production of appreciation at any moment j. This effect on subsequent capital is an investment return from producing appreciation at j that reduces the cost of production at j. The authors show that the marginal cost at j equals[7]

$$\pi_{mj} = \frac{w\partial t_{mj}}{\partial M_j} - A_j = \frac{w}{MP_{t_{mj}}} - A_j \tag{24}$$

where *w* is the wage rate, assumed to be constant over time, and $A_j$ is the effect of addiction that measures the value of saving in future time inputs from the effect of the production M in j on subsequent music capital. $A_j$ is positive as long as music is beneficially addictive, and tends to decline as j increases, approaching 0 as j approaches n. The term $w/MP_{tm}$ declines with age for a given time input as long as music capital grows with age. The term $A_j$ may not change so much with age at young ages because the percentage decline in the number of remaining years is small at these ages. Therefore, $\pi_m$ would tend to decline with age at young ages because the effect on the marginal product of the time input would tend to dominate the effect on A. Although $\pi_m$ might not

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can be viewed as involving participants who maximize their utility from a stable set of preferences and accumulate an optimal amount of information and other inputs in a variety of markets…".

7 For a full demonstration see Stigler and Becker (1997), notes 5, page 79.

always decline at other ages, the authors assume that it declines continuously with age. If $\pi_z$ does not depend on age, the relative price of music appreciation would decline with age. Then by equation (21) the relative consumption of music appreciation would rise with age. The relative consumption of music appreciation rises with exposure not because tastes shift in favour of music, but because its shadow price falls as skill and experience in the appreciation of music are acquired with exposure.

A final remark is important when dealing with model that introduce consumption capital and additional goods in the utility function. In these cases there is a problem in reaching the steady state and a complex dynamics of the pattern of growth also emerges. In the case of cultural consumption and growth present in the model of Scandizzo (1993), these are likely candidates for chaotic behaviour.[8] In his work the author considers the aggregate problem of growth of a country, where aggregate consumption contains a relevant cultural component. The utility function, which measures the general well-being, contains as its argument consumption per capita, while the production function depends on capital and labour. Consumption and accumulation are linked by a double feedback: on the one hand the household production function *f(c,k)* contains consumption as a variable input. If people consume more it does not determine an equally lower level of investment, since consumption contributes to accumulation through a positive marginal productivity of the function *f(c,k)*. On the other hand, a higher level of the capital stock k will also imply a higher level of immediate utility, thanks to the complementarity with consumption. The growth problem is characterized by two opposite forces. On the production side, the fact that consumption can contribute to accumulation favours a higher degree of consumption today. On the utility side, the fact that capital increases utility both directly and as a complement to consumption, implies that a higher degree of accumulation is more desirable. Growth now proceeds indefinitely, since the fall in the marginal productivity of capital may be offset by the positive effect exhibited by capital growth upon both the enjoyment drawn from consumption $U_{ck}$ and the productivity of consumption in the production function $f_{ck}$. Analysing the growth path of consumption and capital, what Scandizzo finds is that the possibility of benefiting from cultural consumption and accumulation introduces an element of inherent instability in the growth paths. The instability generated is of such a complex type that it can be classified under the heading of chaotic behaviour. The author so concludes that cultural consumption and growth appear as likely candidates for chaotic behaviour.

In a work of Boyer (1978), the properties of the growth path have been analysed. The model is an attempt to introduce a limited form of non-additivity utility function. It represents a habit-forming model where the utility of a given level of present consumption depends on the past level of consumption. Boyer demonstrates that there is a sole solution, since the objective function is strictly concave and the feasible set is convex. He finds also that if the starting point is the steady state, it is optimal to stay there. A striking difference between the sufficient conditions for a steady state in the usual no-habit-forming model and the conditions derived by the author is that the steady state is not necessarily reached when the rate of interest is equal to the adjusted rate of time preferences. If consumption per capita is not at its steady state level then the rate of interest will depart from its steady state value and later come back to it; similarly for consumption. It is therefore possible to experience cycles in consumption, investment, capital and the interest rate.

Finally, Ryder and Heal (1973) analyse the properties of optimal growth with intertemporally dependent preferences. They derive, under non-satiation, the existence of a unique optimal steady state, the modified golden rule, but the optimal path need not be monotonic and

[8] Chaotic behaviour is a relatively complex behaviour which is strictly governed by a mathematical algorithm, but is nonetheless unpredictable due to sensitivity to initial conditions. So, although *in principle* it is possible to predict how a system will behave to an arbitrary level of precision, *in practice* it is not possible to find the initial starting point of the system accurately enough to be able to predict in detail what will happen beyond a short period of time.

moreover may oscillate around the optimal steady state. Under satiation, multiple steady states may exist.

What emerges in different analyses is then the possibility, in the presence of a model that considers addictional goods and capital consumption, that there will be unstable steady states, chaotic behaviours or the existence of multiple steady states.

## 2. The model

### 2.1 A short introduction to the model

By using the concepts of addictional goods and consumption capital illustrated in paragraph 1, we now formulate a model of altruism, virtuous capital and happiness. Our aim is to show what the implications of altruism and accumulation of virtuous capital are on individual behaviour. Specially, we are interested in those consumption choices of material goods, monetary donations and altruistic behaviour that lead individuals to reach the maximum level of happiness and satisfaction. In order to reach this objective we solve a dynamic optimization problem through *the maximum principle.*

The reference model is that of Scandizzo (1993), which analyses the problem of consumption of cultural goods as opposed to the consumption of material goods. He defines cultural goods as "…those artefacts or services which through the display of beauty , knowledge or memories of the past, offer an enduring experience….". In consuming cultural goods, individuals accumulate a stock of immaterial wealth, which incorporates knowledge, habit formation and some forms of human capital. The main characteristic of cultural goods is the fact that the satisfaction individuals obtain from their fruition in a certain period of time is positively correlated with the quantity they consumed in all previous periods. He also underlines three relevant aspects in the formation of culture:1) the consumption of cultural goods requires some leisure time; 2) to have some stock of culture allows a higher combination of cultural consumption and leisure to produce culture; 3) the complementarity between leisure and human capital and the difficulty of raising the productivity of cultural goods can cause a lower production of cultural goods.

Scandizzo assumes that the representative consumer maximizes over his lifetime the present value of a utility function *U(.)*, whose level depends on the level of five variables: material goods consumption ($c_1$), cultural goods consumption ($c_2$), leisure (e), the stock of material wealth (W), the stock of cultural experience (A). The utility function takes the form

$$V = \int_0^T e^{-rt}[U(c_1(t), c_2(t), e(t), W(t), A(t))]dt \tag{25}$$

where r is the rate of interest, t denotes time and A(t) is the product of a *household production function* that combines cultural goods, culture and leisure according to the law of motion

$$\frac{dA}{dt} = \overset{\circ}{A} = h(c_2(t), A(t), e(t)) - \theta A(t) \tag{26}$$

where θ represents the degree of time decay of the human capital created by A(t). What is new in this model with respect to the traditional literature on economic growth is that consumption here may contribute to a capital accumulation. This is the concept of *household production function*, as described above. The consumption of cultural goods adds to the stock of knowledge and turns into accumulation of human capital. Accumulated human capital now becomes an argument of the utility function.

After a dynamic maximization, it can be seen that the growth of cultural goods consumption over time responds to four factors:1) the replacement of the stock of culture lost through forgetfulness or by delaying its production because of time preference; 2) the positive external effects for utility and production caused by accumulation of culture; 3) the fact that culture is a substitute for cultural goods consumption both in the household production function and in general utility; 4) the necessity to satisfy the budget constraint by compensating for the simultaneous rise or fall of the stock of material wealth. Moreover cultural goods consumption will accelerate in response to a higher accumulation of culture, while cultural goods production will generally decelerate as the larger amount of leisure needed will be obtained in part at the expense of labour allocated to cultural goods.

From the conditions of growth of cultural goods and leisure it emerges that neither cultural goods nor leisure may ever reach the steady state, since the stimulus to their consumption is compounded, while most of the opponent stimuli tend to decrease with time. In the steady state conditions, on the contrary, there is a quite different situation, the level of consumption is entirely independent of the complementarities between culture and cultural goods, while the direct effects on utility prevail.

While Scandizzo introduces cultural goods consumption as a control variable, we focus our attention on monetary donations. The hypothesis that people are altruistic has a long tradition in economics[9]and it has been used to explain charitable donations and the voluntary provision of public goods. An altruistic[10] view of a person is a view that values positively what is good for another person. Giving is an unconditional action of a person, purposefully favourable in some way to another and costly in some way for the actor. However, apart from this cost, the giver may benefit from other effects of his action and give for this reason. An altruistic view leads its holder to give when this person finds that the valuable consequences more than compensate the costs for him. An altruist is someone who holds altruistic views, or someone who gives for this reason. Altruism means both the existence of altruistic views and the resulting givings[11].

[9] Many economists, including Adam Smith (1759), Gary Becker (1974), Paul Samuelson (1993), Amartya Sen (1993), pointed out that people often care for the well-being of others and this may have important economic consequences. Other works analysing individual's daily behaviours (Andreoni (1995), Croson (1996), Rabin (1993)), confirm the idea that individuals are not merely selfish in their lives.

[10] The word altruism, derived from the French autre, in its turn derived from the Latin alter, was coined by Auguste Comte, in order to describe the ethical doctrine of positivism. He believed that individuals had a moral obligation to serve the interest of others or the "greater good" of humanity. Comte says that "(the) social point of view cannot tolerate the notion of rights, for such notion rests on individualism. We are born under a load of obligations of every kind, to our predecessors, to our successors, to our contemporaries. After our birth these obligations increase or accumulate, for it is some time before we can return any service.... This (to live for others), the definitive formula of human morality, gives a direct sanction exclusively to our instincts of benevolence, the common source of happiness and duty. (Man must serve) Humanity, who we are entirely." As the name of the ethical doctrine is altruism, doing what the ethical doctrine prescribes has also come to be referred to by the term altruism, serving others through placing their interests above one's own.

[11] Kolm (2006) describes various types of altruistic views, according to their causes or reasons. According to these reasons it is possible to identify two categories of altruism: *hedonistic or natural altruism*, and *normative altruism.* The

It is also possible to formalize altruistic behaviour in the following way: a person is altruistic if the partial derivatives of his utility function $U(x_1 \ldots\ldots x_n)$, with respect to $x_1 \ldots\ldots x_n$, are strictly positive, which means that his utility increases with the well-being of other people.

After having defined altruistic behaviour, we want to underline one important feature of monetary donations. They are a kind of *addictional goods*. Satisfaction the consumer obtains from such a fruition over a period of time is positively related to the quantity consumed in the previous period. Actually, the more an individual behaves altruistically, the higher will be the stock of *virtuous capital* accumulated. People will then be more and more able to appreciate benefits deriving from monetary donations and participation in voluntary organizations. Virtuous capital is produced by a function that depends on the accumulated stock of virtue and on monetary donations

$$V(t) = f(V(t), d(t))$$

According to this expression, donations may be considered a factor of production. This idea is synthesized with the concept of the *household production function* introduced by Becker(1964, 1965). In our model, in particular, the stock of virtue V(t) is the product of a *household production function*, that combines the existing stock of virtue V, and monetary donations d, according to the law of motion

$$\frac{dV}{dt} = \overset{\circ}{V}(t) = \delta V(t) + \alpha d(t)$$

Donations, in particular, by adding to the stock of virtuous capital, turn into an accumulation of this capital. The accumulated virtuous capital itself becomes an argument of the utility function. Then, the utility provided by a given level of monetary donations changes in both quantity and quality. Productivity of *altruism* may also be increasing.

Finally, donations and accumulation of virtuous capital are linked to each other by a double feedback. On the one hand, the household production function $V(t) = f(V(t), d(t))$ contains d as a variable input. A higher level of donations contributes to higher level of virtuous capital. On the other hand, a higher level of $V$ determines a higher utility immediately and, through its complementarity with donations and material goods consumption, will favour higher material goods consumption and monetary donations today. However a trade-off exists between monetary donations and material goods consumption, given the wealth constraint.

Given this short description of the model's main features, we proceed in following paragraphs with its formalization and with an empirical work.

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social psychological phenomena of affection, sympathy, empathy, emotional contagion, fellow feeling, compassion and pity make a person feel happy or sad as a consequence of the happiness or pain, or the bad or good situation of another person. They induce *natural or hedonistic altruism*. On the other hand, *normative altruism* is induced by moral intuition, non-moral social norms, or various applications of reason or rationality. Moral intuition and moral reason induce two kinds of *moral altruism*.

**2.2 A model of altruism and happiness: a single consumer**

This model aims to examine the problem of resources allocation between monetary donations and consumption of material goods. We assume that the "representative consumer" maximizes over his lifetime the present value of a utility function U(.), the level of which depends on the value of three variables: 1) consumption of material goods (x); 2) donations of money (d); 3) the stock of virtue $V$ (immaterial capital). In mathematical terms, this implies the following formulation

$$MaxS = \int_0^T e^{-rt}\left[U(x(t),d(t),V(t))\right]d(t) \tag{27}$$

where

$$\frac{\partial U}{\partial x}>0,\frac{\partial^2 U}{\partial x^2}<0,\frac{\partial U}{\partial d}>0,\frac{\partial^2 U}{\partial d^2}<0,\frac{\partial^2 U}{\partial x\partial d}=0,\frac{\partial U}{\partial V}>0,\frac{\partial U^2}{\partial V^2}<0,\frac{\partial U^2}{\partial x\partial V}>0,\frac{\partial U^2}{\partial d\partial V}>0,\frac{\partial V}{\partial d}>0$$

and r is the rate of time preference.

Fixed $V\in\Re$ and that $V(t)=V,\forall t,$ then assumptions $\frac{\partial^2 U}{\partial x^2}<0,\frac{\partial^2 U}{\partial d^2}<0$ assure us the strict concavity of the utility function. As can be seen in equation (27), individuals' utility can be raised through material goods consumption and through donations, that are neither complementary nor substitute ($\frac{\partial^2 U}{\partial x\partial d}=0$). Given the wealth constraint, they are mutually exclusive.

A higher level of $V$ determines a higher utility both directly and indirectly through complementarity with material goods consumption, ($\frac{\partial U^2}{\partial x\partial V}>0$), which means that a higher stock of virtue increases the capacity of individuals of obtaining satisfaction from material consumption, and monetary donations ($\frac{\partial U^2}{\partial d\partial V}>0$). This last conditions represents the characteristics of addictional goods: their consumption depends on the previous level of consumption, as for cultural goods consumption (Scandizzo 1992, 1993, Stigler and Becker 1977).

Donations also raise utility through two channels: directly, as they are an argument in the utility function, and indirectly through their contribution to the accumulation of virtue. Individuals who have altruistic behaviour accumulate a stock of knowledge, skills and consciousness, the *virtuous capital* that allows them to be altruistic in the future. As a consequence, the higher the stock of virtue, the higher actual and future donations and the higher the utility individuals obtain from donations.

Other conditions that have to be respected are the following: x(t) and d(t) take only null or positive values $x(t)\geq 0$, $d(t)\geq 0$. V(t) assumes only positive values, $V(t)>0$, while W(t) is subject to the following constraint

$$0\leq x(t)+d(t)-\rho W(t)\leq W(t) \tag{28}$$

where ρ is the rate of return of material wealth.

The stock of virtuous capital V(t) is the product of a *household production function* that combines donations and virtue according to the law of motion

$$\frac{dV}{dt} = \overset{\circ}{V}(t) = \delta V(t) + \alpha d(t) \tag{29}$$

In equation (29), $\overset{\circ}{V}$ (t) represents the time derivative of virtue, δ is the effectiveness of the stock of virtue and α is the donations productivity in accumulating virtue.

The intertemporal budget constraint can be written as the law of motion for the stock of material wealth

$$\frac{dW}{dt} = \overset{\circ}{W}(t) = \rho W(t) - x(t) - d(t) \tag{30}$$

where $\overset{\circ}{W}$ (t) is the time derivative of *W(t)*, *W* is the stock of material wealth and ρ is the rate of return on material wealth.

Equations (27), (29) and (30) describe an optimal control problem, with control variables x(t) and d(t), and state variables V(t) and W(t)

$$MaxS = \int_0^T e^{-rt}\left[U(x(t), d(t), V(t))\right]dt$$

subject to

$$\overset{\circ}{V}(t) = \delta V(t) + \alpha d(t)$$
$$\overset{\circ}{W}(t) = \rho W(t) - x(t) - d(t)$$
$$V(0)=V_0=K>0$$
$$W(0)=W_0=A\geq 0$$
and $0 \leq x(t) + d(t) - \rho W(t) \leq W(t)$
$x \in [0, W], d \in [0, W]$ for all $t \in [0, T]$

The solution can be found by applying Pontryagin's maximum principle. By the standard definition the Hamiltonian takes the form

$$H = e^{-rt}[U(x(t), d(t), V(t)] + \mu_1(\delta V(t) + \alpha d(t)) + \mu_2(\rho W(t) - x(t) - d(t)) \tag{31}$$

The Hamiltonian corresponds to the discounted value of the utility obtained from consumption goods, donations, virtuous capital and material wealth. $\mu_1$ and $\mu_2$ represent the costate variables that are similar to Lagrange multipliers. They measure the shadow price of an associated state variable, which means that along the optimum path costate variables evaluate the increase in the utility function corresponding to an increase in per capita virtue *V* and material wealth *W* .

In order to solve the maximization problem, we need also the concepts of current-value Lagrange multipliers, which can be derived as follow

$$\lambda_1 = \mu_1 e^{rt} \tag{32}$$

$$\lambda_2 = \mu_2 e^{rt} \tag{33}$$

Then the current-value Hamiltonian, denoted by $H_c$, can be written as

$$H_c = He^{rt} = [U(x(t), d(t), V(t)] + \lambda_1(\delta V(t) + \alpha d(t)) + \lambda_2(\rho W(t) - x(t) - d(t)) \tag{34}$$

We use equation (34) in order to derive first order conditions for the maximization of S (equation 27), under the dynamic constraints on the stock of virtue (equation 29) and wealth (equation 30)

$$\frac{\partial H}{\partial x} = U_x - \lambda_2(t) = 0 \tag{35}$$

$$\frac{\partial H}{\partial d} = U_d + \lambda_1(t)\alpha - \lambda_2(t) = 0 \tag{36}$$

where $U_x = \partial U / \partial x$ and $U_d = \partial U / \partial d$

and

$$\overset{\circ}{\lambda}_1(t) = -\frac{\partial H_c}{\partial V} = (r - \delta)\lambda_1 - U_v \tag{37}$$

$$\overset{\circ}{\lambda}_2(t) = -\frac{\partial H}{\partial W} = (r - \rho)\lambda_2 \tag{38}$$

where $U_v = \partial U / \partial v$

We also add the following transversality conditions

$$\lambda_1(T) = 0, \lambda_2(T) = 0 \tag{39}$$

Equation (35) states that the marginal utility of wealth, along the optimal path, is equal to the marginal utility of consumption of material goods. Equation (36) indicates that, along the optimal path, the marginal utility of donations plus the value at shadow price of donations productivity in accumulating virtue has to equal the marginal utility of wealth. Equation (37) states the law of growth of the shadow price of virtue accumulation along the optimal path. This shadow price grows with the net rate of time preference (i.e. the rate of time preference minus the effectiveness of stock of virtue) minus the marginal utility of virtue. Equation (38) indicates that the shadow price of material wealth accumulation grows over time with the rate of time preference and reduces with the rate of return of wealth.

Differentiating expressions (35) and (36) with respect to time yields

$$U_{xx}\overset{\circ}{x}(t) + U_{xd}\overset{\circ}{d}(t) + U_{xv}\overset{\circ}{V}(t) - \overset{\circ}{\lambda}_2(t) = 0 \tag{40}$$

$$U_{dd}\,\overset{\circ}{d}(t)+U_{dx}\,\overset{\circ}{x}(t)+U_{dv}\,\overset{\circ}{V}(t)+\alpha\,\overset{\circ}{\lambda}_1(t)-\overset{\circ}{\lambda}_2(t)=0 \tag{41}$$

Given $U_{xd}=0$, substituting (29), (35) and (38) into (40) and solving for $\frac{\overset{\circ}{x}}{x}$ we obtain

$$\frac{\overset{\circ}{x}}{x}=\frac{1}{\eta_x}[(\rho-r)+\frac{U_{xV}}{U_x}(\delta V(t)+\alpha d(t))] \tag{42}$$

where $\eta_x=-(U_{xx}/U_x)x$ is the elasticity of marginal utility of consumption goods. Then, by substituting equations (29), (36), (37), (38) into (41) we obtain

$$\frac{\overset{\circ}{d}}{d}=\frac{1}{\eta_d}[(\delta-r)+(\rho-\delta)\frac{U_x}{U_d}+\frac{U_{dv}}{U_d}(\delta V(t)+\alpha d(t))-\alpha\frac{U_v}{U_d}] \tag{43}$$

where $\eta_d=-(U_{dd}/U_d)d$ is the elasticity of marginal utility of donations, $U_x/U_d$ is the marginal rate of substitution between donations and consumption goods, $U_V/U_d$ is the marginal rate of substitution between donations and virtue.

Expression (42) shows that the growth of consumption goods along the optimal path increases with the rate of return on wealth, while it reduces with the rate of time preferences. According to the wealth constraint (28), higher $\rho$ determines greater possibilities to consume material goods. Complementarity between $x$ and $V$ produces a positive effect on the rate of growth of $x$, while marginal utility of material goods reduces as the quantity of $x$ rises. Expression (43) indicates that the growth of donation is inversely related to the net rate of time preference and the marginal utility of virtuous capital. Complementarity between $d$ and $V$ produces a positive effect on the rate of growth of donations[12].

## 3. The database

In order to empirically test our model, we use data drawn from a database created through a questionnaire submitted from November 2006 to January 2007 to a random sample of Italian population. The survey contains more than 40 questions, aimed at collecting: a) personal data; b) attitude toward risk; c) measures and concepts of happiness; d) objective and subjective events and desires; e) working conditions; f) relational goods; g) perceived level of inflation and unemployment.

[12] After having derived the dynamic equations (42) and (43) for x(t) and d(t), we ask ourselves whether there is a steady state solution at a common rate. We equal equation (54) to equation (55) as in the following expression:

$$\frac{1}{\eta_x}[(\rho-r)+\frac{U_{xV}}{U_x}(\delta V(t)+\alpha d(t))]=\frac{1}{\eta_d}[(\delta-r)+(\rho-\delta)\frac{U_x}{U_d}+\frac{U_{dv}}{U_d}(\delta V(t)+\alpha d(t))-\alpha\frac{U_v}{U_d}]$$

After long and tedious computation we find that *the equilibrium does exists* (for instance one can use $U(x,d,V)=e^{hV}(x^{\frac{h-1}{k}}+d^{\frac{h-1}{k}})$) where $h,k\in\Re$.

A description of the variables employed in our empirical work is presented in table 28. In order to calculate the first dependent variable, the natural logarithm of material goods consumption, we use question 6[13], through which we ask people their annual income, and question 7[14], that is the percentage of income used for material goods consumption. Then we calculate the natural logarithm of the value obtained. For the second dependent variable, the natural logarithm of monetary donations, we ask respondents directly the amount of their annual income devoted to donations (question 37)[15]. The rate of return on material wealth is derived by asking people (question 10)[16] what kind of investment they prefer among treasury bills, bonds and equity, and associating to treasury bills the 2007 annual average return, to bonds the 2007 annual average return as calculated by Mediobanca, and to equity the MSCI Italy return for 2005. The rate of time preference is calculated by asking people (question 11)[17] to choose between a lower amount of money today or a higher amount of money available in one year. In the survey there is no question about virtuous capital. Nevertheless, we ask people if they are active in volunteering activities. We can use this information to proxy virtuous capital by a binomial variable which takes on value 1 if individuals are active in volunteering associations, 0 otherwise. By using the data collected through question 1[18], we calculated the age of interviewed individuals, while the sex, the level of annual income and the number of children are obtained through questions 3[19], 6[20] and 9[21] respectively. The variable number of children is a dummy variable which takes on value 0 when the respondent has not children and value 1 when the interviewed person has 1 or more children.

## 4. The empirical analysis

In order to test the results obtained in our model, we estimate the differential equations of material goods consumption and donations. According to the stochastic utility model, we assume that material consumption and donations are linearly dependent on a vector of social and economical characteristics, and on a stochastic term with zero mean.

An estimable version of (42) and (43) can then be written as:

$$\ln x_i = \gamma + \beta_1(\rho_i - r_i) + \beta_2 V_i + \beta_3 d_i + \beta_4 age + \beta_5 sex + \beta_6 inc + \beta_7 child + \varepsilon_i \qquad (44)$$

[13] Question 6 of the survey is : "Could you indicate your annual class of income, choosing among the following: 1) less than 10000; 2) 10000-20000; 3) 20000-30000;4) 30000-40000;5) more than 40000".
[14] Question 7 of the survey is : "Could you indicate the percentage values of your income used for the following types of expenditure: 1) house; 2) food; 3) free time and holidays; 4) transport; 5) clothes 6) internet, computer and phone; 7) other. In particular, we consider as material goods consumption the bundle of goods indicated by ISTAT (2008), that corresponds to the expenditure for house, food, free time and holidays, transport, clothes, internet, computer and phone.
[15] Question 37 of the survey is :" How much of your annual income is devoted to donations? 1) nothing; 2) up to 100 euro; 3) from 100 to 1000 euro; 4) more than 1000 euro.
[16] Question 10 of the survey is :" Which is your preferred type of investment? 1)treasury bills; 2) bonds; 3) equity. (2008), that corresponds to the expenditure for house, food, free time and holidays, transport, clothes, internet, computer and phone.
[16] Question 37 of the survey is :" How much of your annual income is devoted to donations? 1) nothing; 2) up to 100 euro; 3) from 100 to 1000 euro; 4) more than 1000 euro.

[17] Question 11 of the survey is :"Imagine that you win 10000 euro in a lttery, but you can receive the money only in one year. One of your friend propose you to buy now your ticket for 9300 euro. Will you accept? If your answer is no, what amount of money would you ask for your ticket?"
[18] Question 1 of the survey is :"Please indicate your year of birth"
[19] Question 3 of the survey is :"Please indicate your sex"
[20] Question 6 of the survey is :"Please indicate your class of level of annual income"
[21] Question 9 of the survey is " Do you have children? If yes, how many?"

$$\ln d_i = \lambda + \theta_1 r_i + \theta_2 \rho_i + \theta_3 V_i + \theta_4 age + \theta_5 sex + \theta_6 inc + \theta_7 child + \mu_i \qquad (45)$$

In order to correct for heteroscedasticity, we also assume that the Stochastic disturbance has a standard deviation that linearly depends on the same socio-economic characteristics.

Equations (44) and (45) assume that the individual consumption and utility parameters from the sample differ from each other because of socio-economic differences, as well as idiosyncratically (through the random disturbance). According to our model, we expect to find $\beta_1>0$, $\beta_2>0$, $\beta_3<0$, $\theta_2>0$. Moreover in equation (44), $\gamma=\ln(x_i,t-1)$, $\rho_i$ is the rate of return of material wealth, $r_i$ is the rate of time preference of individual i, $V_i$ is the stock of virtue, $d_i$ are the donations of individuals i and $\varepsilon_i$ are the residuals. In equation (45)

$$\lambda = \ln d_{i,t-1} + \frac{1}{\eta_d}\delta - \frac{U_x}{U_d}\delta - \alpha\frac{U_V}{U_d}$$

and $\mu_i$ are the residuals of person i.

We thus control for age, sex, level of annual income and presence of children that can affect both material goods consumption and monetary donations.

Equation (44) implies the constraint that the coefficient of $\rho$ equals the coefficient of $-r$, thus an unconstrained version of equation (44) has been estimated previously and a Wald test does not reject the constraint. Afterwards the constrained version has been estimated. In table 29 the result of the regression is reported. A unit increase in the difference between the rate of return of wealth and the rate of time preference determines a 0,933% increase in *material goods consumption*. A unit increase of monetary donations produces a 0,011% decrease of material goods consumption. The meaning of virtuous capital coefficient can be interpreted as suggested by Gujarati (2004, pg.321). By calculating the following expression

$$(e^{\hat{\beta}} - 1) * 100 \rightarrow (e^{0,189} - 1) * 100 = 20,80\%$$

it is possible to find that individuals engaged in volunteering activities have a median consumption 20,8% higher than those individuals not involved in such activities[22]. These results are in line with the predictions of our model, because an increase in wealth determines both a higher consumption of material goods, and an increase in virtuous capital.

Table 30 shows the results of equation (45). A unit increase of returns on wealth positively affects the *monetary donations* by 0,6%. As before, because virtuous capital is a dummy variable we can interpret the coefficient of virtuous capital by calculating the following expression

$$(e^{\hat{\beta}} - 1) * 100 \rightarrow (e^{0,354} - 1) * 100 = 42,47\%$$

Individuals engaged in volunteering have median donations 42,47% higher than with respect to individuals who are not involved in these activities. All variables are significant. Even for this equation the results of our model are confirmed. An increase in wealth determines a higher level of monetary donations, such as an increase in virtuous capital.

[22] This interpretation is possible because we estimate semilogarithmic equations and the regressors involved are relative to dummy variables.

## 5. Concluding remarks

This work is an original attempt to study altruistic behaviour by introducing three explanatory variables into the utility function: the consumption of material goods, monetary donations and the stock of virtuous capital. Altruistic behaviour appears to be characterized by few but sharply distinguishing features: the nature of addictional goods in the form of monetary donations and the complementarity between the stock of virtue and donations. Both these characteristics induce individuals to become increasingly altruistic.

The model proved to be not well-behaved. We tried to work it out by applying Pontryagin's maximum principle and obtained a system of differential equations. However, given the features for monetary donations illustrated above, we found that donations rise over time, by responding to the following needs: i) to replace the stock of virtuous capital lost by delaying its production because of time preferences; ii) to account for the fact that donations are a substitute for material goods consumption in generating utility; iii) to take advantage of the positive externalities that accumulating virtues generates for utility. In particular, these are inversely related to the net rate of time preference and to the marginal utility of virtuous capital, while the complementarity between $d$ and $V$ determines the opposite effect on the rate of growth of donations. With respect to consumption of material goods, it grows along the optimal path with the rate of return on wealth, while it decreases with the rate of time preferences. A higher $\rho$ determines greater possibilities to consume material goods. Complementarity between $x$ and $V$ produces a positive effect on the rate of growth of $x$, while the marginal utility of material goods falls as the quantity of $x$ rises. The growth of monetary donations along the optimal path is inversely related to the net rate of time preference and the marginal utility of virtuous capital. Complementarity between donations and virtuous capital produces a positive effect on the rate of growth of donations.

The empirical analysis confirms the intuitions of our model. With respect to the equation of material goods consumption, an increase in the difference between the rate of return on wealth and the rate of time preferences determines an increase in the natural logarithm of material goods consumption, while a raise in monetary donations produces a decrease in material goods consumption. We also find that in the median term people engaged in volunteering activities have a consumption higher by 20,8% with respect to individuals not involved in those kinds of activities. With regard to the equation of monetary donations, we find that an increase in the rate of return on wealth positively affects donations. In the median term people engaged in volunteering activities make donations higher by 42,47% with respect to individuals not involved in those activities.

In conclusion, it is important to note some limitations of the study. As stated before, the model did not appear to be well-behaved and proved quite complex to solve. It is certainly possible to proceed with further computations and add other intuitions about the behaviour of the variables. Moreover it could be useful to find other measures of the stock of virtue.

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## Tables

Table 1: Description of variables

| VARIABLES | QUESTIONS | MEAN VALUE | STANDARD DEVIATION |
|---|---|---|---|
| **$lnx_i$,** natural logarithm of material goods consumption | Question 7 of the questionnaire ask respondents to specify which percentage of their annual income is devoted to: house, food, travel and free time, transports, clothes, internet and mobile phones, others. By using the definition of bundle of consumption goods given by ISTAT (2008), we sum as material goods the following items: house, food, clothes, transport, travel and free time, internet and mobile phones. percentage of annual income spent in material goods | 9,66446219 | 0,69679913 |
| **$lnd_i$,** natural logarithm of monetary donations | Question 37 of the questionnaire asks respondents to specify which amount of their annual income is devoted to donations. In this case we take the natural logarithm. | 4,45431941 | 0,97941055 |
| **$\rho_i$**, rate of return on wealth | Question 10 of the survey ask people what kind of investment they prefer among treasury bills, bonds and equity. We can associate to treasury bills the 2007 annual average return, to bonds the 2007 annual average return as calculated by Mediobanca, and to equity the MSCI Italy return for 2005. | 0,05221587 | 0,01766862 |
| **$r_i$,** rate of time preferences | Question 11of the survey asks people to choose between a lower amount of money today or a higher amount of money available in one year. This allows us to derive the rate of time preference | 0,03567264 | 0,04152211 |
| **$V_i$,** Virtuous capital | Since there is no data on the stock of virtue, we proxy it by a binomial variable which takes on value 1 if individuals are active in volunteering associations, 0 otherwise. The presence of this activity means that individuals have an altruistic attitude, so they accumulate virtuous capital. It has no meaning to evaluate mena andstandard deviation. | . | . |
| **$d_i$,** monetary donations | Question 37 of the questionnaire ask respondents to specify which amount of their annual income is devoted to donations, by proposing four alternatives: 1) nothing; 2) up to 100 euros; 3) from 100 to 1000 euros; 4) more than 1000 euros. | 89,5833333 | 164,510485 |

Table 2: Empirical estimate of equation (44):Dependent variable: log of material goods consumption

| | NATURAL LOGARITHM OF MATERIAL GOODS CONSUMPTION |
|---|---|
| Return on wealth-Rate of time preference | 0,933*(0,525) |
| Income | 0,290**(0,13) |
| Age | 0,011*(0,006) |
| Sex | 0,301**(0,117) |
| Donations | -0,011*(0,005) |
| Virtuous capital | 0,189**(0,114) |
| Presence of children | 0,249**(0,116) |
| Constant | 8,435***(0,225) |
| ***p<.01<br>**p<0.05<br>*p<0.1 | R-squared=0.1248 |

Standard Error in parenthesis

Table 3: Empirical estimate of equation (45):Dependent variable: log of donations

| | **NATURAL LOGARITHM OF DONATIONS** |
|---|---|
| Rate of time preference | -0,027*(0,016) |
| Return on wealth | 0,519**(0,257) |
| Virtuous capital | 0,354***(0,128) |
| Age | 0,002(0,001) |
| Sex | 0,000(0,000) |
| Presence of children | 0,011(0,023) |
| Annual Income | 0,062*(0,009) |
| Constant | 4,632***(0,695) |
| ***p<.0.1<br>**p<0.05<br>*p<0.1 | R-squared=0.10587 |

Standard Error in parenthesis