Risks For The Long Run And The Real Exchange Rate[*]

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Abstract

We propose an equilibrium model that can explain a wide range of international finance puzzles, including the high correlation of international stock markets despite the lack of correlation of fundamentals. We conduct an empirical analysis of our model, which combines cross-country-correlated long-run risk with Epstein and Zin (1989) preferences, using US and UK data and show that it successfully reconciles international prices and quantities, thereby solving the international equity premium puzzle. These results provide evidence suggesting a link between common long-run growth perspectives and exchange rate movements.

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1 Introduction

Since Backus and Smith (1993), a long strand of the literature has documented the tension between international prices and quantities. As noted among others by Backus, Foresi, and Telmer (1996) and Lustig and Verdelhan (2006), if markets are complete, the growth of the exchange rate would equal the difference of the stochastic discount factors for foreign currency– and dollar–denominated assets. Based on this insight, Brandt, Cochrane, and Santa-Clara (2006) conclude that the modest degree of international correlation of consumption growth observed in a large cross-section of countries should result in larger fluctuations in exchange rate growth rates.[1] By extension, they suggest that given the degree of volatility of the US dollar that we observe in the data, there should be a larger correlation of stochastic discount factors across countries.

The Hansen and Jagannathan (1991) bound on the volatility of the logarithm of the stochastic discount factor is in the order of roughly 40% per annum in major industrialized countries. The standard deviation of the log-growth of the exchange rate between the same countries is typically between 11% and 15%. These numbers imply a lower bound on the correlation of the stochastic discount factors of approximately 0.95.[2]

If we turn our attention to quantities the results change substantially. If investors have CRRA preferences, their stochastic discount factors will be proportional to consumption growth rates. It is well documented that consumption is poorly correlated across countries at annual or higher frequencies. If we were to set the coefficient of risk aversion to a number as high as 30 to reconcile the low volatility of consumption growth rates with highly volatile stochastic discount factors, exchange rates would have to fluctuate between 15 and 30 times more than what we observe in the data. Furthermore, the low correlations of international quantities seem to be at odds also with the actual degree of co-movement of stock markets. Given the low international correlation of consumption and dividend cash flows, why are international

[1]This relies on the assumption that agents have constant relative risk-aversion preferences.

[2]This claim is based on the aforementioned condition: $\log\left(\frac{e_{t+1}}{e_t}\right) = \log M^f_{t+1} - \log M^h_{t+1}$, where e denotes the exchange rate, and M^h and M^f are the stochastic discount factors of the home and foreign country, respectively. See Brandt, Cochrane, and Santa-Clara (2006) for a detailed discussion of this equation.

returns so correlated?

This dichotomy of prices and quantities strikes us as an important unresolved puzzle in international finance. We like to think of this as the *international equity premium puzzle*. In a one-country model, consumption growth does not *vary* enough to explain the excess return over the risk-free rate. In a two-country model, consumption growth does not *co-vary* enough to track movements in the exchange rate and returns.

In this paper we propose an equilibrium model that can simultaneously reproduce a number of findings regarding international quantities and prices. Indeed, in our model, consumption growth rates prove to be as volatile, as autocorrelated, and as cross-country-correlated as they are in the data. Furthermore, the volatility of the exchange rate growth resembles the one that we observe for the US dollar, and the currencies of most countries. International stock markets are as highly correlated as they should be and the equity premium puzzle is solved within each country.

Our model features an additional source of risk that is highly correlated across countries. This risk enters as a small, slowly moving, predictable component of consumption growth that is consistent with the empirical evidence showing consumption to be close to a random walk. We refer to these components as the long-run risks. This addition changes the joint conditional distribution of consumption growth rates quite a bit. In particular, while over a short horizon the long-run component explains only a very small share of consumption volatility, over a longer horizon the long-run risks are responsible for most of the fluctuations and the co-movements of domestic and foreign consumption.

We combine this specification with the assumption that home and foreign investors have Epstein and Zin (1989) preferences, as in Bansal and Yaron (2004). In this economy, agents are extremely sensitive to uncertainty about the long-run perspectives of the economy. If the long-run components are highly correlated across countries, then stochastic discount factors will be too. Ultimately, exchange rates need to fluctuate much less to prevent international arbitrage opportunities. Furthermore, by shifting the focus on investors' aversion to intertemporal risk, we do not need to rely on high risk-aversion coefficients to justify large equity premia in our economy.[3] This enables us to track the low volatility of exchange rate movements despite the low cross-country correlation of consumption growth in the short-run.

Our theoretical analysis, however, goes beyond solving the exchange rate volatility puzzle raised by Brandt, Cochrane, and Santa-Clara (2006). Our framework has also precise implications for the international correlation of the returns. It has always been hard to reconcile the almost complete lack of correlation of fundamentals between countries with the high degree of international stock markets co-movements. Ammer and Mei (1996) and Bansal and Lundblad (2002) point out that common long-run perspectives should be responsible for stock markets moving together to the extent that we see in the data. We incorporate their insight in a consumption-based equilibrium framework and show that the long-run co-movement in consumption and dividend dynamics is able to replicate the joint behavior of international asset returns and exchange rate.

When we calibrate our model according to US and UK data, our model is also able to replicate the unconditional average of foreign risk-adjusted returns. This means that it can account for the international investment opportunity set of both a US-based investor and a UK-based investor. If agents care about the temporal distribution of risk, small but long-lasting and highly cross-border correlated shocks to cash flows will lead to large price movements in both countries. This means that international returns are more correlated.

After analyzing our theoretical model under the extreme assumption of perfectly correlated long-run news, we address the following empirical question: is it possible to provide a direct estimate of the low-frequency components in consumption and make the case that long-run growth rate prospects look similar across countries?

In order to answer this question, we adopt the view that the long-run movements of consumption trend can be identified by means of predictive variables. Similarly to Bansal, Kiku, and Yaron (2006) and Colacito and Croce (2008a), we project consumption growth on lagged values of price-dividend ratio, consumption-output ratio, and consumption growth itself, to measure long-run risks. Applying this methodology to US and UK data separately, we conclude that

[3] We analyze the cross-country intertemporal composition of consumption risk focusing only on non-durable goods and services. The interaction between US non-durable consumption risk and currency returns is analyzed by Lustig and Verdelhan (2007) in the context of a model with recursive preferences.

the predictable components of consumption growth rates are highly persistent and that their correlation increases over time, just as the volatility of exchange rate growth decreases. The latter result reinforces our conjecture regarding the link between common long-run growth perspectives and exchange rate movements.[4]

Using this measure of long-run risks, we can also test the moment restrictions resulting from our model and estimate preference parameters. Results confirm that a reasonable range of values for the intertemporal elasticity of substitution should be $[0.25, 3]$ and that a coefficient of risk aversion smaller than 10 can account for both the domestic and international version of the equity premium puzzle. When we focus only on the post-Bretton Woods sample, the typical result is that our model cannot be rejected.

One of the novelties of our empirical work is that we identify long-run consumption risk using information from domestic financial markets and better assess the impact that such news has on currency markets. This enables our model to bridge part of the gap between international prices and quantities.

The paper is organized as follows. In section 2, we propose a simple model that we can both solve and calibrate. This provides a useful instrument to show the internal transmission mechanism of the economy. In sections 3 and 4, we demonstrate the ability of the model to account for a large number of international moments. The focus of section 5 is on the estimation of the model. Section 6 concludes the paper, with a discussion of the potential extensions of the model and a summary of the main findings.

2 Setup of the economy

We analyze two economies that we denote as home (h) and foreign (f). These two economies each feature a representative consumer with Epstein and Zin (1989) preferences

$$U_t^i = \left\{(1-\delta^i)(C_t^i)^{\frac{1-\gamma^i}{\theta^i}} + \delta^i \left[E_t\left[(U_{t+1}^i)^{1-\gamma^i}\right]\right]^{\frac{1}{\theta^i}}\right\}^{\frac{\theta^i}{1-\gamma^i}}, \forall i \in \{h, f\}$$

[4]Among others Lustig, Verdelhan, and Roussanov (2008) have suggested that common risk factors across countries may play a crucial role in explaining the cross-sectional properties of currency markets.

where γ^i is the coefficient of risk aversion and $\theta^i = \frac{1-\gamma^i}{1-1/\psi^i}$ implicitly defines the intertemporal elasticity of substitution ψ^i. There are two country specific goods in the economy. To simplify the setup, we impose that preferences are such that there is complete home bias, meaning that the representative consumer in each country is willing to consume only the good that she is endowed with. Markets are complete. An equilibrium of this economy exists, in which each country behaves as in autarky both for consumption and asset holdings.

The logarithm of the pricing kernel m_{t+1}^i is a stochastic process that depends on both log-consumption growth, Δc_t^i, and the log-return on the asset that pays consumption, $r_{c,t+1}^i$:

$$m_{t+1}^i = \theta^i \log \delta^i - \frac{\theta^i}{\psi^i} \Delta c_t^i + (\theta^i - 1) r_{c,t+1}^i, \forall i \in \{h, f\} \tag{2.1}$$

We complete the system by specifying exogenous laws of motion for consumption and dividend growth rates as follows:[5]

$$\begin{aligned} \Delta c_t^i &= \mu_c^i + x_{t-1}^i + \varepsilon_{c,t}^i \\ \Delta d_t^i &= \mu_d^i + \lambda^i x_{t-1}^i + \varepsilon_{d,t}^i \\ x_t^i &= \rho_x^i x_{t-1}^i + \varepsilon_{x,t}^i, \quad \forall i = \{h, f\} \\ \xi_t &\sim_{i.i.d.} N(0, \Sigma) \end{aligned} \tag{2.2}$$

where $\xi_t = [\ \varepsilon_{c,t}^h \quad \varepsilon_{d,t}^h \quad \varepsilon_{x,t}^h \quad \varepsilon_{c,t}^f \quad \varepsilon_{d,t}^f \quad \varepsilon_{x,t}^f\ ]$ is the vector of shocks of the economy.

In this section, the two economies are assumed to be symmetric, having the same preference and transition laws parameters. Furthermore, we assume that the shocks $\varepsilon_{c,t}^i$, $\varepsilon_{d,t}^i$, and $\varepsilon_{x,t}^i$ are mean zero $i.i.d.$ normally distributed within each country with volatilities σ, $\varphi_d\sigma$, and $\varphi_e\sigma$, respectively. The shocks are allowed to be cross-country correlated. We shall denote ρ_c^{hf} as the correlation between $\varepsilon_{c,t}^h$ and $\varepsilon_{c,t}^f$; ρ_d^{hf} as the correlation between $\varepsilon_{d,t}^h$ and $\varepsilon_{d,t}^f$; and ρ_x^{hf} as the correlation between $\varepsilon_{x,t}^h$ and $\varepsilon_{x,t}^f$. All other correlations are set to zero.

[5]We can think of the dynamics in (2.3)–(2.5) as the outcome of post-trade allocations. Colacito (2006a) provides general equilibrium foundations that ensure the existence of an exchange rate in this economy. Following the literature on the dynamics of risk-sensitive allocations by Kan (1995), Dumas, Uppal, and Wang (2000) and Anderson (2005), Colacito and Croce (2008a) derive laws of motion of consumption that are very similar to those used in this paper.

The following system is obtained as a first-order linear approximation of the model:[6]

$$\begin{aligned}
m_{t+1}^i &= \log\delta - \frac{1}{\psi}x_t^i + \delta\frac{1-\gamma\psi}{\psi\left(1-\rho_x\delta\right)}\varepsilon_{x,t+1}^i - \gamma\varepsilon_{c,t+1}^i \qquad (2.3)\\
\Delta e_{t+1} &= m_{t+1}^f - m_{t+1}^h\\
v_{d,t}^i &= \overline{v}_d + \frac{\lambda-\frac{1}{\Psi}}{1-\rho_x\delta}x_t^i\\
r_{d,t+1}^i &= \overline{r}_d + \frac{1}{\psi}x_t^i + \delta\frac{\lambda-\frac{1}{\psi}}{1-\rho_x\delta}\varepsilon_{x,t+1}^i + \varepsilon_{d,t+1}^i\\
r_{f,t+1}^i &= \overline{r}_f + \frac{1}{\psi}x_t^i, \qquad \forall i\in\{h,f\}
\end{aligned}$$

where $v_{d,t}^i$ is the price-dividend ratio, $r_{f,t+1}^i$ is the log–risk-free rate, and $\overline{r}_j$ is the average return on asset j. In what follows, we shall refer to Δe_{t+1} as the exchange rate growth or as the depreciation rate. Given the system (2.3), the following two propositions can be stated.

Proposition 1. *For a given choice of parameters and provided that $\rho_x^{h,f} \geq \rho_c^{h,f}$, the lowest cross-country correlation of the stochastic discount factors is achieved for*

$$\psi = \frac{1}{\gamma}\widetilde{\delta} \quad , \quad \rho_x = 0 \quad , \quad \rho_x^{hf} = \rho_c^{hf}$$

where $\widetilde{\delta} = \frac{1-2\rho_x\delta+\delta^2}{\delta^2(1-\rho_x^2)}$. Furthermore, if $\rho_x^{h,f} > \rho_c^{h,f}$, then $(\psi, \rho_x) = \left(\frac{1}{\gamma}\widetilde{\delta}, 0\right)$ is the unique minimizer.

Proof. See Appendix. □

Proposition 2. *For a given choice of parameters, the lowest volatility of the depreciation rate is achieved for $\rho_x^{hf} = 1$.*

Proof. See Appendix. □

Propositions 1 and 2 highlight three crucial ingredients necessary to obtain highly correlated stochastic discount factors together with low-volatility exchange rates. First of all we must

[6] See the Colacito and Croce (2008b) for details.

uncouple the tight relationship between risk-aversion and intertemporal elasticity of substitution, that constant relative risk aversion preferences would otherwise impose. Indeed, as the subjective discount factor δ approaches unity, the minimum correlation of stochastic discount factors is obtained for a value of intertemporal elasticity of substitution that is arbitrarily close to the reciprocal of the coefficient of risk aversion.[7] This is independent of the calibration of the rest of the model. Hence it does not seem implausible to view the contribution of Brandt, Cochrane, and Santa-Clara (2006) as the positing of a lower bound on the correlation of stochastic discount factors in a consumption-based asset pricing model. Proposition 1 is silent about agents' preference for early or late resolution of uncertainty.[8] In the next section we show that any departure from constant relative risk-aversion preferences yields highly correlated stochastic discount factors. If the intertemporal elasticity of substitution is too small, a counterfactual outcome of infinitely volatile returns results, as suggested by the system of equations (2.3).

Proposition 2 captures the role of the predictable components of consumption growth rates. The intuition is straightforward. Epstein and Zin (1989) preferences introduced an extra term in the formula for the stochastic discount factors in the two countries, in the form of the return that pays the consumption bundle at each period. The price of this asset reflects the total expected flow of dividends, in our case consumption, to which investors are entitled. Since we model consumption as having a predictable component, it is natural to conclude that this term has an impact on the return of the asset that is proportional to its persistence. By appropriately raising the correlation of the x_t's, we increase the correlation of the returns on the consumption assets and ultimately the correlation of stochastic discount factors. The exchange rate, therefore, needs to fluctuate less in order to prevent international arbitrage opportunities. In the next section we demonstrate that it is the combination of all the three elements identified in propositions 1 and 2 that enables our model to deliver the desired results.

[7]This is the case when the model is calibrated to a monthly or quarterly decision problem.

[8]Epstein and Zin (1989) argue that an agent prefers early resolution of uncertainty if $\psi > 1/\gamma$, while late resolution is preferred if $\psi < 1/\gamma$.

3 Reconciling international prices and quantities

In this section we describe the ability of the model to account for a wide variety of international finance puzzles. We will first focus on a symmetric calibration exercise, that allows us to explain in detail the internal transmission mechanism of the model. We then relax this simplifying assumption and prove that the model retains its explanatory range and power. As in the previous section, the shocks $\varepsilon^i_{c,t}$, $\varepsilon^i_{d,t}$, and $\varepsilon^i_{x,t}$ are mean zero $i.i.d.$ normally distributed within each country.

[INSERT TABLE 1 ABOUT HERE]

3.1 Calibration

The structure of our two parallel economies mimics those discussed by Bansal and Yaron (2004) and Bansal, Gallant, and Tauchen (2007); most of the coefficients used in our analysis are either estimated or calibrated in those papers. In table 1 we report our baseline calibration.

The subjective discount factor is set to 0.998, because our model describes a monthly decision problem. In terms of proposition 1, this means that the correlation of stochastic discount factors is minimized for values of $\psi \approx 1/\gamma$. We set the coefficient of risk aversion γ equal to 4.25, which is relatively low compared to both the number commonly found in the equity premium puzzle literature and the coefficient proposed by Brandt, Cochrane, and Santa-Clara (2006). The intertemporal elasticity of substitution-ψ is equal to 2 as estimated by Bansal, Gallant, and Tauchen (2007).

As far as the calibration of the parameters of the laws of motion of consumption growth is concerned, our goal is to reproduce the average behavior of consumption growth in the United States and in the United Kingdom. We motivate in detail this specific choice of countries in the next section. Here we anticipate that we need to work with countries that display international financial openness, and that have long time series for consumption and dividends.

We set $\rho_x = 0.987$, which is the value estimated by Bansal, Gallant, and Tauchen (2007) and is slightly higher than the 0.979 calibrated by Bansal and Yaron (2004). The standard deviation of the shock to the predictable components, $\varphi_e \sigma$, is extremely small compared to that of the shock to consumption growth, σ, allowing the latter to be the main determinant of the volatility of consumption growth over the short-run. This is in the spirit of a large part of the literature, which models consumption growth as an almost *i.i.d.* process (see, among others, Tallarini (2000)). The standard deviation of consumption growth implied by our choice of parameters is approximatively 2.4% in annualized terms. This falls in-between the US and UK's average growth of per capita consumption of nondurables and services from 1970 to 1998.

The dividends processes require the calibration of four additional parameters, which we have selected with the goal of matching the first two moments of dividend growth rates, their cross-country correlation, and their leverage as the average of actual US and UK data. By design the variance of dividend growth explained by its predictable component is very small, in the order of 3%.

The choice of the correlation coefficients is driven by the need to match key features of the data. We set ρ_x^{hf} to 1, as suggested by proposition 2, to keep the volatility of the depreciation rate to about 11%-12%.[9] The cross-country correlation of the shocks to consumption, ρ_c^{hf}, is chosen so as to obtain a correlation of consumption growth in the order of 0.3. We show below that our results are robust to the relaxation of the assumption of symmetric calibration.

We begin by justifying the assumptions that we made about the consumption processes on the grounds of the data. In most of the existing literature researchers have typically struggled to find conclusive evidence of the consumption process that we adopt in our model, when the focus is on the US only.[10] In figure 1 we report the estimated spectra and coherence of US and UK consumption growth (continuous lines) along with the theoretical periodograms and the 95% confidence intervals (dashed lines) under the alternative assumptions that consumption growth is *i.i.d.* (left panel) and that it has a small predictable component (right panel).[11] The

[9] In section 4 we relax this assumption and we work with two estimated long-run components that are not perfectly correlated.

[10] See for example Hansen, Heaton, and Li (2008).

[11] See Colacito and Croce (2008b) for details.

confidence intervals are computed by simulating 1000 independent samples according to the model in eq. (2.2) with the calibration of table 1, except for σ_c^h, σ_c^f, and ρ_c^{hf}, which are set to match the same sample variance and covariance of the actual US and UK series. Results, shown in figure 1, indicate that the two alternative assumptions appear equally likely, as the estimated spectra and coherence typically lie within the 95% confidence intervals. An examination of international quantity data, hence, does not enable us to identify the low-frequency components of consumption growth rates. This is why our empirical analysis focuses on the joint behavior of international prices and quantities.[12]

[INSERT FIGURE 1 ABOUT HERE]

3.2 Correlation of stochastic discount factors

In the previous section we have discussed how the combination of long-run risks, Epstein-Zin preferences and highly cross-country-correlated x^h and x^f drive the results. In figure 2 we depict this finding with regard to the correlation of stochastic discount factors. Using the system of equations in (2.3), we investigate the variation of this correlation with the coefficient of intertemporal substitution. In each of the two panels, the dark line is drawn according to our baseline calibration. In light of proposition 1, we are not surprised that the minimum of this graph falls in the vicinity of $\psi \approx 1/\gamma$. By moving away from $1/\gamma$ in either direction, there is a sharp increase in the correlation of stochastic discount factors. However, as stressed in the discussion of the two propositions, increasing the correlation of stochastic discount factors by lowering the intertemporal elasticity of substitution would have the drawback of making the risk-free rates too volatile. It is also for this reason that in our baseline calibration, we set $\psi = 2$, which leaves us with a correlation that is well above 0.9, as the price data seem to suggest. Interestingly, we do not need to take a stand on the intertemporal elasticity of substitution being greater or smaller than 1, in order to obtain

[12] In an earlier version of this paper, we use the Kalman filter to obtain a recursive characterization of the likelihood function starting from the state space representation for consumption growth rates. The estimated parameters are typically characterized by very large confidence intervals, and this finding remains true even when a large cross-section of countries is employed. These results are available upon request.

highly correlated stochastic discount factors. Any ψ larger than 0.5 would yield stochastic discount factors for the two countries that are at least 80% correlated (see figure 2), provided that the remaining coefficients are calibrated as in table 1.

This comment deserves particular attention, given that the value of the intertemporal elasticity of substitution has been the subject of a long-lasting debate in the literature. Ignoring the presence of stochastic volatility, Hall (1988) and Lustig and Van Nieuwerburgh (2006) estimate this number to be below unity. Guvenen (2006) points out that limited participation in the stock market is able to produce findings consistent with both capital and consumption fluctuations, as long as most of the wealth is held by a small fraction of the population with a high elasticity of intertemporal substitution. Attanasio and Weber (1989) use the *Family Expenditure Survey* to document an intertemporal elasticity of substitution in excess of one in the UK. Our paper follows the tradition of the long-run risk literature in calibrating $\psi = 2$ with the objective of appropriately describing the first two moments of returns, as will be discussed in the next sections.

If we perturb our baseline calibration, a lower correlation of the long-run shocks would noticeably decrease the correlation of stochastic discount factors (figure 2 upper panel). This effect can be seen in the reduced form of m_t^h and m_t^f in the system of equations in (2.3). Indeed, the higher the cross-country correlation of shocks of any nature, the more the stochastic discount factors will tend to co-move. In contrast to most international finance models, we are able to achieve this result by keeping the unconditional international correlation of real quantities to the low level that we observe in the data. We will test this assumption against the data in a later section.

The reduction of the persistence of the predictable components of consumption growth rates in the two countries also affects the success of our analysis (figure 2, lower panel). The reason is simple: for the small shock to the trend of consumption to matter, it must be long-lasting. If investors care about the intertemporal distribution of risk, as they do when they have Epstein and Zin preferences, a small but persistent shock will have an important effect on the discounting of risky payoffs. Therefore it is crucial that in both countries investors perceive these risks as near-permanent changes in the average growth of consumption.

We do not increase the persistence of the predictable components in our model because by doing this we would increase (at an increasing rate) the share of consumption growth variance that is explained by its predictable component, eventually contradicting the empirical evidence that consumption growth is almost an *i.i.d.* process. In the figures, we do not report the behavior of the variables of interest when changing the coefficient of risk aversion; rather we refer to proposition 1 to explain what would happen: given that the subjective discount factor is close to 1, lowering (increasing) the coefficient of risk aversion would move the minimum of the correlation of stochastic discount factors to the right (left) in figure 2.

[INSERT FIGURE 2 ABOUT HERE]

3.3 Volatility of exchange rates fluctuations

Exchange rate movements are intimately related to investors' degree of risk aversion. By keeping the correlation of long-run shocks to a high level, the dynamics of the exchange rates are governed mostly by short-run shocks. Indeed, in the limiting case of our benchmark calibration, short-run shocks are solely responsible for exchange rate fluctuations:

$$\Delta e_{t+1} = \gamma(\varepsilon^h_{c,t+1} - \varepsilon^f_{c,t+1}).$$

This condition is identical to the one that we would obtain from employing time-additive CRRA preferences.[13] Given the low correlation of short-run shocks to consumption, good news in the home country is likely to come with no news in the foreign country. Because of the consumption-smoothing desire of investors, the demand of home securities will increase. The foreign agent is also eager to invest in the home country, as this looks like a good insurance opportunity. The increased foreign demand of domestic securities thus let the exchange rate appreciate by a factor that is proportional to agents' risk aversion.

In figure 3 we plot the volatility of exchange rate growth against the coefficient of risk aver-

[13] In all subsequent sections, we relax the assumption of perfect correlation of long-run risks. This results in the long-run shocks also appearing in the dynamics of exchange rate movements.

sion. The two horizontal dashed lines represent the region within which the volatility level of the depreciation rate typically falls for major industrialized countries. In our baseline calibration we set $\gamma = 4.25$ which yields a volatility that is well within the region of interest, as documented by the dark lines in the two panels of figure 3. Increasing the risk aversion of the representative consumers of the two countries to the high levels required by the equity premium puzzle literature would push the volatility to counterfactual levels–similar to those that motivated Brandt, Cochrane, and Santa-Clara (2006) to postulate the existence of a puzzle. In this case, our calibration again proves itself crucial.

Reducing the correlation of the long-run shocks reduces the correlation of stochastic discount factors, as we documented before, and increases the exchange rate fluctuations. Unless agents are made less risk averse, we are left with exchange rates that are too volatile at the calibrated level of risk aversion (figure 3, upper panel). The high exchange rate volatility can be offset by decreasing the persistence of the predictable component (figure 3, lower panel). In this way, the volatility of the stochastic discount factors falls more than their international correlation. However, the evidence reported in figure 2 prevents us from taking this action, because a less persistent long-run component implies a correlation of stochastic discount factors that is too small. Furthermore, less persistence produces less-volatile stochastic discount factors and, hence, counterfactual implications for asset prices.

[INSERT FIGURE 3 ABOUT HERE]

3.4 Other moments of international financial markets

In table 2 we show that our baseline calibration is able to match key features of international financial markets. In comparing our results to the data, we have assumed the United States to be the home country and the United Kingdom to be the foreign country. We have already documented the ability of the model to produce highly correlated stochastic discount factors and exchange rates that are as volatile as they are in the data. Our symmetric calibration implies that the average growth of the US dollar vis-à-vis the British pound should be zero, which is not too far from the 1.3% sample average obtained in the post-Bretton Woods era.

[INSERT TABLE 2 ABOUT HERE]

Bansal and Yaron (2004) have shown that by focusing on investors' concerns about the intertemporal distribution of risk, it is possible to increase the premium that equities command over the risk-free rate. Since high average excess returns are a commonly found in major industrialized countries, extending this logic enables us to reproduce the first two moments of stock markets' excess returns beyond the United States. Indeed, the model produces equity premia and risk-free rates whose averages and standard deviations are precisely in line with those observed in the data.

We push our analysis one step further, to demonstrate that the model can also match the average correlation of excess returns in the two countries. This is accomplished in spite of the low calibrated correlation of dividend growth rates. We regard this positive finding as further validation of the existence of highly cross-country-correlated predictable components of consumption and dividend growth. When cash flow growth rates are *i.i.d.*, the price-dividend ratios are constant and returns are as cross-country correlated as are the growth of the dividends—a counterfactual implication. By employing Epstein and Zin preferences and by allowing a small but highly persistent and highly cross-country-correlated component be also responsible for the dynamics of consumption and risky cash flows, we unveil an all-new layer of stock markets co-movements.[14]

When bad news about long-run growth perspectives hits one country, the other country is hit by the bad news as well. The more that investors are averse to the intertemporal unfolding of risk, the more prices will respond to this news. As the shocks are highly correlated, this implies a large degree of co-movement in asset prices and returns. This limits international risk-sharing opportunities, at least as far as long-run risks are concerned.

[14] Bansal and Lundblad (2002) have shown that a high correlation in the long-run dynamics of dividends can generate high co-movements in international stock market prices. Their result is obtained in the context of a model in which equity premia evolve according to the CAPM.

Our model seems to produce excess correlation of real risk-free rates. The system of equations (2.3) shows that the approximate solution for the risk-free rates in this economy is a linear function of the predictable component of consumption growth alone. As a consequence, the correlation of real risk-free rates will entirely reflect the perfect correlation of the x's. However, an asymmetric calibration in which long-run shocks are still perfectly correlated, but the persistence of long-run risks is slightly different across countries, produces the right amount of risk-free rate correlation. We provide empirical support of this finding in the next section.[15]

Up to this point we have focused on returns expressed in local units, r^{US} for the US and r^{UK} for the UK. In rows 10 through 12 of table 2 we report the statistics for returns expressed in domestic units. For a US-based investor, foreign returns expressed in domestic units are $r^{UK}_{t+1}+\Delta e_{t+1}$. Similarly, for a UK-based investor returns equal $r^{US}_{t+1}-\Delta e_{t+1}$. Focusing on returns expressed in domestic units is a necessary step in order to study the model's implications for the international investment opportunity sets of both a US- and a UK-based investor.

The international investment opportunity set obtained using our symmetric calibration falls between those that the data suggest for a US- and a UK-based investor (figure 4). This is an important result, especially considering the risk-return tradeoff of exchange rate movements: given that currency depreciations and appreciations carry a small premium and are extremely volatile, one might expect that the domestic investor would have little incentive to invest abroad. However, foreign assets turn out to be attractive for two crucial reasons. First, exchange rate movements appear to be uncorrelated with foreign asset price fluctuations. Second, the cross-country correlation of returns expressed in local units is about 60%. As table 2 suggests, our model succeeds in replicating both of these pieces of empirical evidence, which enables us to account for foreign Sharpe ratios expressed in domestic units and for the optimal reward-to-variability ratio that may be achieved by US- and UK-based investors.

[INSERT FIGURE 4 ABOUT HERE]

[15] As an alternative, Colacito (2006b) proposes a generalization of the framework analyzed in this paper that includes more than one predictable component of consumption growth in each country, while maintaining a symmetric calibration.

In the lower part of table 2 we report some statistics for consumption and dividend growth rates for the US and UK that confirm that our calibration is consistent overall with the empirical evidence. Our assumption of symmetric calibration results in some small discrepancies between the predictions of our model and what is suggested by the data. In the next section we relax this assumption, which yields a better model fit.

3.5 Relaxing the assumption of symmetric calibration

We now investigate what happens when we no longer require that home and foreign countries share the same parameterization. It turns out that given this relaxation our model will retain its ability to account for both exchange rates and investment opportunity sets while more closely replicating the international correlations of assets.

The sets of coefficients for the two countries and the correlations are chosen to minimize a GMM criterion on a total of 46 moment conditions. We think of this procedure as a formal data-driven calibration exercise. The 16 estimated coefficients are reported in table 3. We set the cross country correlation of the long run shocks to 1 as our preliminary empirical results suggest that this coefficient might lie on the boundary of the parameter space.[16] Doing this allows us to use the standard asymptotic results of Hansen (1982). The technical appendix by Colacito and Croce (2008b) reports the details of the estimation.

Due to the analytical difficulty of computing the quarterly counterparts of the moments of interest using a model that is specified at a monthly frequency, we follow Hansen and Sargent (2006) in assuming that model and data share the same frequency. We employ the same data set used by Campbell (2003), which comprises the same sample (1970.1 to 1998.4) used by Brandt, Cochrane, and Santa-Clara (2006). Quantity data are quarterly observations on private consumption of nondurables and services obtained from NIPA for the US and from IMF's *International Financial Statistics* (IFS). We deflated the data using the Consumer Price Index provided by CRSP for the United States and by IFS for the UK. The real US dollar-

[16] Andrews (2002) warns that when the true parameter is on the boundary, GMM estimators are not asymptotically normally distributed. To avoid having to deal with non-standard distributions of our estimates, we set $\rho_x^{hf} = 1$ and interpret our results as a conditional GMM.

British pound exchange rate is derived as the ratio of the price index in UK currency and the Morgan Stanley Capital International (MSCI) price index in dollar terms. The risk-free rates are the T-bill rates from CRSP for the US and the rate at which 91-day T-bills are allotted in the UK. Stock market returns and dividends are for value-weighted portfolios obtained from CRSP for the US and from Morgan Stanley Capital Perspective for the UK.

For comparison, in table 3 we show the results obtained when only consumption data are used to estimate the parameters governing consumption growth dynamics. The point estimates are very much in line with our previous calibration. The standard errors associated with these point estimates, however, confirm the finding of figure 1: quantities alone are not enough to assign tight confidence intervals to the dynamics of the low-frequency components of the consumption growth rates.

[INSERT TABLE 3 ABOUT HERE]

Even though these characteristics of consumption cannot be clearly identified from quantity data alone, exchange rates and asset returns help in this task. Indeed, adding price restrictions helps in tightening the confidence intervals of the point estimates of these sets of parameters around the values that we specified in the theoretical analysis of the model (see the last two columns of table 3). The predictable components of consumption growth are very close to unit roots, and yet the data tell us that these dynamics can be consistent with almost $i.i.d.$ growth rates. The reason has to do with the fact that the volatilities of the shocks to the time-varying trend, $\varphi_e \sigma$, of consumption are very small, and hence the low persistence of growth rates is almost entirely driven by the short-run shocks. One key finding of table 3 is that by using prices information it is possible to conclude that the parameters governing the dynamics of the predictable components of consumption growth rates are all significantly different from zero. This finding is apparent when comparing point estimates and standard errors of σ_x^h and σ_x^f across the procedure that involves only consumption data and the one that also uses pricing restrictions. In regard to the preference parameters, the two intertemporal elasticities of substitution are significantly larger than the reciprocal of the coefficients of risk aversion, as this is necessary in order to explain the high equity premium in the US and in the UK.

Even though the parameters in the two countries are no longer set to the same values and the model is specified at a quarterly frequency, it is still possible to account for the moderate degree of volatility of the depreciation rate of the US dollar (see the lower part of table 3). In fact, it is possible to improve even further the ability of the model to account for the degree of cross-country correlation of asset returns. In particular, the correlation of excess returns is now exactly equal to that observed in the data, and the model even manages to uncouple the perfect correlation of risk-free rates obtained in the context of the symmetric calibration exercise. Sharpe ratios expressed in local units are extremely close to the data, as the model now features the small amount of US dollar appreciation that we documented in the previous section.

All these findings add a degree of robustness to the results previously described and highlight the ability of the model to deliver highly correlated stochastic discount factors. Indeed, this correlation is 0.972 using the point estimates of our GMM. This leads us to wonder whether it is possible to estimate the time series of x_t^h and x_t^f, which have so much to do with our main result. In the next section of the paper we investigate this matter in greater detail.

4 Estimating international long-run risks

The task of measuring long-run risks from consumption data is made even harder in small samples. In essence, this is the message of the first three columns of table 3. However, the ability to estimate the predictable components of consumption growth at each point in time plays a crucial role in the testing of our model against the data. In this section we explore one way to construct an observable time series of long-run risks. We focus on the US and the UK for a number of reasons both practical and historical. First of all, the identification of a highly persistent time series requires as long a sample as possible. From this stand-point, US and UK are probably the best examples of countries with a long history of accurate consumption data. Second, the US and UK have had an excellent tradition of financial integration at least for a large part of the post-World War II period, as documented by Taylor (1996), Obstfeld (1998), and Quinn (1997) among others. This is important because in assessing the predictions of the model for the dynamics of international asset prices, we need to ensure that US and UK agents considered each other's assets as part of their own investment opportunity sets.

4.1 Long-run risks: A predictive regressions approach

The literature on long-run risks documents that variables, such as lagged price-dividend and consumption-output ratios, risk-free rates, lagged consumption growth, and default premia, should contain a direct measure of long-run risk at each date and state. Based on this prediction, we take the projection of consumption growth on the aforementioned set of variables to be our measure of long-run risks. The literature is not new to this approach: Harvey (1988) was one of the first to suggest the use of price information to predict future consumption. We explore the potential of this procedure by applying it to both the US and the UK. Harvey (1986) suggests that *"using longer term consumption growth rates may reduce the relative size of any errors in the consumption data."* Following this insight we focus on annual consumption growth rates, as they are more likely to contain a better signal and less noise regarding the low frequency component of consumption. This also enables us to obtain longer time series for the US and the UK.

Our common sample covers the years from 1929 to 2006. Data on US consumption of non-durables and services, gross domestic product, and population were collected from the NIPA tables of the Bureau of Economic Analysis. Value-weighted market returns, yields on 3-month T-bills, dividends, and dividend yields for the US are from CRSP. CPI inflation and the spread between BAA and AAA corporate bonds (our measure of the default premium) were obtained from the web-site of the Federal Reserve of St. Louis. The UK consumption series for the years 1963–2006 were obtained from the UK Office of National Accounts. For earlier data, we referred to Mitchell (1979). Long time series for the UK gross domestic product, FTSE returns, yields on three-month T-bills, dividend yields, population, and CPI inflation were obtained from the website of Global Financial Data. All variables have been de-meaned.[17]

[INSERT TABLE 4 ABOUT HERE]

In table 4 we report the results of our analysis. The second and fourth rows of the table shows the estimated coefficients of the regressions:[18]

$$\Delta c_t^i \quad = \quad \beta_0 \Delta c_{t-1}^i + \beta_1 pd_{t-1}^i + \beta_2 \Delta cy_{t-1}^i + \beta_3 default_{t-1}^i + \varepsilon_{c,t}^i \tag{4.4}$$

and

$$\widehat{x}_t^i \quad = \quad \rho \widehat{x}_{t-1}^i + \varepsilon_{x,t}^i$$

where $\widehat{x}_t^i = \widehat{\beta}_0 \Delta c_{t-1}^i + \widehat{\beta}_1 pd_{t-1}^i + \widehat{\beta}_2 \Delta cy_{t-1}^i + \widehat{\beta}_3 default_{t-1}^i, \forall i \in \{US, UK\}$. Standard errors are Newey-West adjusted for heteroskedasticity. Our results suggest that approximately 30% to 40% of consumption growth variance can be predicted at an annual frequency and that the persistence of the predictable component of consumption growth is quite high. By comparison, the monthly equivalents of our estimates of ρ_x^{US} and ρ_x^{UK} are 0.964 and 0.978, respectively.

[INSERT FIGURE 5 ABOUT HERE]

In figure 5 we plot the time series of $\widehat{x}_t^{US}$ and $\widehat{x}_t^{UK}$. A number of interesting facts emerge from this picture. First, the correlation between the predictable components of consumption growth seems to change over time. Indeed, it ranges from a historical low value of 0.11 from the beginning of the sample until 1941, to an all-time high level of 0.90 in the last twenty years. Second, there appears to be a clear relationship between the correlation of the predictable components of consumption and the volatility of the growth of the dollar-pound exchange rate. In particular, if we divide the post–Bretton Woods sample into two parts, the lowest exchange

[17] All data are available upon request.

[18] The default premium was not included in the UK regression, because we were unable to find a long time series for it.

rate volatility occured in the years in which the correlation of long-run growth prospects was the highest. This finding provides further empirical validation of our theoretical model.

The third and fifth rows of table 4 are narrowly focused on the predictive power of price-dividend ratios, a variable that the model suggests should contain a direct measure of the x's. Specifically, the results are based on the following regressions:

$$\Delta c_t^i = \alpha p d_{t-1}^i + \varepsilon_{c,t}^i \tag{4.5}$$

and

$$\widehat{x}_t^i = \rho \widehat{x}_{t-1}^i + \varepsilon_{x,t}^i$$

where $\widehat{x}_t^i = \widehat{\alpha} p d_{t-1}^i$, $\forall i \in \{US, UK\}$. The results of these regressions are in line with those discussed above. The use of price-dividend ratios alone, however, buys us additional persistence in the dynamics of the predictable components of consumption growth, an important ingredient in our theoretical analysis. The results in figure 6 confirm the high degree of co-movement of the low frequency components of consumption in the US and the UK in the last thirty years.

[INSERT FIGURE 6 ABOUT HERE]

4.2 Long-run risk, consumption and Euler equations

In this section we use the set of predictive variables for consumption outlined in the previous section to test a set of moment restrictions resulting from our model (equations (2.1)-(2.2)). In particular, we focus on the Euler equations for domestic and foreign stock market returns and risk-free rates for US- and UK-based investors and on the first two moments of exchange rate growth.

[INSERT TABLES 5 AND 6 ABOUT HERE]

In tables 5 and 6 we report the results of two alternative approaches. In the first case, we take the time series of long-run risks as exogenously given and use the moment conditions to estimate the preference parameters of the two countries' investors. Secondly, we jointly estimate the set of preference parameters and coefficients governing the dynamics of the predictable components of consumption growth in the US and the UK. In the latter case, we also include the set of least-squares orthogonality conditions to identify the additional parameters of the model.

We adopt a continuous GMM estimator in which the optimal weighting matrix is continuously updated as the numerical values of the parameters are changed during the minimization of the criterion function. Hansen, Heaton, and Yaron (1996) show that this estimator is better behaved in small samples. Specifically, the test for over-identifying restrictions is more reliable and the parameters' estimates display less median bias and performs better than under other GMM-based methods. This seems to be particularly true for data sampled at an annual frequency.[19] In tables 5 and 6 we show the results of our estimation over various subsamples and using different sets of predictive variables to filter long-run consumption risks.

The model is overwhelmingly rejected by the data when the analysis is conducted on the entire sample for years 1931–2006. This finding is robust across our set of predictive variables and estimation procedures. It is also not surprising. As exchange rates abandon the regime of nominal rigidities imposed by the Bretton Woods system, the degree of volatility of this series changes dramatically. Furthermore, an extensive literature documents that the degree of financial markets' openness between the US and the UK increased only after the late 1960's. Therefore, a focus on the entire sample may be inappropriate when it comes to testing the ability of our model to reproduce the average volatility of the exchange rate and the validity of some of the Euler equations.[20]

The model seems to provide a better fit of the data when long-run risks and preference parameters are jointly estimated. This is the result of a higher persistence of the two predictable

[19] A more detailed description of the econometric procedure can be found in the Colacito and Croce (2008b).

[20] See Quinn (1997) for an empirical study of the degree of financial openness of OECD countries. See Colacito and Croce (2008a) for a theoretical model providing a link between international capital mobility and exchange rate movements across regimes in the context of a model with recursive preferences.

components that makes the stochastic discount factors more volatile and more correlated across countries. When we apply this estimation approach to the post–Bretton Woods sample, the model is not rejected by the data.

The estimated preference parameters are in line with the numbers that we suggested in the earlier sections. The intertemporal elasticity of substitution is typically in excess of one, and the coefficient of risk aversion is quite low compared to the rest of the literature on the equity premium puzzle. The model seems to do particularly well in the 1974-2006 subsample. This is probably a reflection of the fact that the transition to the new exchange regime lasted for a few years.

It appears that price-dividend ratios alone are enough in order to provide an excellent fit of the data. This may be explained by the correlation between returns and the stochastic discount factors. When all predictive variables are employed, a large share of the dynamics of the x's is governed by lagged consumption growth. For this reason, the stochastic discount factors are poorly correlated with returns, a problem typical of most consumption-based asset pricing models. In this case, the high risk-aversion coefficients required in order to explain the high historical equity premium have the undesired effect of causing excessive volatility in the exchange rate growth. On the other hand, the fluctuations of the price dividend ratios are by definition highly correlated with returns. Using the observed shocks to the price-dividend ratios as a measure of the long-run news thus provides a better fit of the Euler equations with reasonable preference parameters. The high implied international correlation of the predictable components then rounds out the explanation of the exchange rate fluctuations.

Overall, we regard the results of this analysis as successful, as they provide evidence of the predictability of consumption growth that can be used to explain key properties of international asset prices. In the next section, we show the benefits of including dividends information as well.

4.3 Long-run risk, dividends and returns

In this section we further investigate the dynamics of returns and exchange rates. Specifically, we implement a GMM estimation that compares the returns and exchange rates implied by our model with those observed in the data. The idea is that the better our model can track the dynamics of actual asset returns and exchange rates, the lower should be the resulting J-statistic.

We proceed in three steps. First, we provide empirical evidence for the presence of a predictable component of dividend growth in the US and in the UK. This is a crucial ingredient in accounting for the dynamics of returns. We then show that the same linear combination of predictive variables has significant forecasting power for both consumption and dividend growth. The exposure of consumption and cash flows to the same long-run risk is fundamental in accounting for the equity premium puzzle in the context of long-run risk models. Finally, we estimate two versions of the econometric model: one in which price-dividend ratios are the only predictors, and one in which the entire set of variables outlined above is employed.

[INSERT TABLE 7 ABOUT HERE]

In the first six columns of table 7 we report the results of least-squares regressions of dividend growth rates on the two alternative sets of predictive variables used in the previous section. The results seem to suggest that there is some predictability of dividend growth in the two countries. The R^2's are lower than those reported in table 4, but most of the estimated coefficients are significantly different from zero. To test whether the predictable component is common to consumption and dividend growth, we use a likelihood-ratio (LR) test. Specifically, we estimate by maximum likelihood the following systems of equations:

$$
\begin{aligned}
\Delta c_t^i &= \beta_0^i \Delta c_{t-1}^i + \beta_1^i pd_{t-1}^i + \beta_2^i \Delta cy_{t-1}^i + \beta_3^i default_{t-1}^i + \varepsilon_{c,t}^i \\
\Delta d_t^i &= \phi_0^i \Delta c_{t-1}^i + \phi_1^i pd_{t-1}^i + \phi_2^i \Delta cy_{t-1}^i + \phi_3^i default_{t-1}^i + \varepsilon_{d,t}^i
\end{aligned}
\tag{4.6}
$$

and

$$
\begin{aligned}
\Delta c_t^i &= \beta_0^i \Delta c_{t-1}^i + \beta_1^i pd_{t-1}^i + \beta_2^i \Delta cy_{t-1}^i + \beta_3^i default_{t-1}^i + \varepsilon_{c,t}^i \\
\Delta d_t^i &= \lambda^i \left(\beta_0^i \Delta c_{t-1}^i + \beta_1^i pd_{t-1}^i + \beta_2^i \Delta cy_{t-1}^i + \beta_3^i default_{t-1}^i \right) + \varepsilon_{d,t}^i
\end{aligned}
\tag{4.7}
$$

for all countries $i = \{US, UK\}$. We regard system (4.6) as the unrestricted version of our model of predictive regressions and system (4.7) as its restricted version. The results of the LR tests for the null that $\phi_j^i = \lambda^i \beta_j^i$, $\forall i = \{US, UK\}$ and $\forall j = \{0, 1, 2, 3\}$ (see table 7) indicate that in both countries the null cannot be rejected at least at the 1% level of significance, which appears to be consistent with our theoretical assumption of a common long-run component in the dynamics of consumption and dividends. On the grounds of this evidence, we assume henceforth that the consumption and cash flows of each of the two countries can be predicted by the same linear combination of variables. This will enable us both to keep our econometric specification close to the theoretical model and to deal with a smaller set of parameters in the estimation.

[INSERT TABLE 8 ABOUT HERE]

In table 8 we report the results of the GMM investigation in which we employed the continuous updating technique discussed in the previous section. We focus here on the mean of domestic and foreign excess returns, the mean of domestic and foreign risk-free rates, and the first two moments of exchange rate growth. We also include the appropriate least-squares orthogonality conditions that are needed to estimate the parameters of the predictive variables.[21]

Our results confirm that when the estimation is conducted on the entire sample, the model is strongly rejected by the data. We argued in the previous section that this should not strike us as a surprise, given that the exchange rate dynamics have been very different across the various regimes of the last century. When we analyze the post–Bretton Woods sample, the

[21] In our log-linear model, the introduction of the mean of the foreign returns in domestic units would make the optimal weighting matrix singular.

results improve significantly. The tests for the over-identifying restrictions cannot reject the model at the 3% level of significance when the price-dividend ratios are the only predictors nor at more than the 10% level of significance when the whole set of predictors is employed. The point estimates of the preference parameters are in line with what we suggested in the outline of our model and they are typically significant.

We regard this result as one of the main contributions of the paper. Standard frictionless consumption-based models are usually rejected by the data and produce unrealistic values for the preference parameters. Our analysis, in contrast, is able to explain the dynamics of international asset returns and exchange rate fluctuations in a framework that is consistent with quantities and that relies on reasonable values for agents' risk aversion.

[INSERT FIGURE 7 ABOUT HERE]

As seen in figure 7, the model is also able to reproduce the time series of excess returns and risk-free rates in the post–Bretton Woods era. When the larger set of predictive variables for consumption is used, the model does a good job of tracking the time behavior of returns and exchange rate movements. Indeed, at our point estimates, the volatility of exchange rate growth is about 16% and hence very close to what we observe in the data. Furthermore, the ability of the model of producing realistic path of international prices and currencies seems to increase over time, especially after the 1984 crises.

5 Concluding remarks

We have shown that by disentangling the intertemporal elasticity of substitution from the reciprocal of the coefficient of risk aversion and by allowing for a persistent and highly cross-country-correlated forecastable component of consumption growth, it is possible to explain a large set of international finance puzzles using a frictionless consumption-based asset pricing model. In our analysis, the dichotomy between international quantities and prices documented by the international finance literature disappears after accounting for the intertemporal composition of consumption risk. Consumption and cash flow dynamics are in line with the data, and at the same time the equity premium puzzle is solved within each country. Exchange rate fluctuations resemble those that we observe for major industrialized countries, and the international investment opportunity set reflects the Sharpe ratios and the degree of correlation suggested by the data.

Future developments of this line of research should address the deeper economic question of the origin of the predictable components of consumption growth in the context of a fully specified production economy. In particular, it will be important to study the predictions of the model for the dynamics of current accounts, portfolio allocations, and international investment flows. We are optimistic that the positive findings of this paper will also hold in more general contexts that have the potential to extend even further the set of properties of this class of models.

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Appendix A. Proof of Propositions

Proof of Proposition 1. For any choice of the parameters that satisfy $\rho_x^i \neq 1$ and $\rho_x^i \delta^i \neq 1$ the following three partial derivatives

$$\frac{\partial corr(m_t^h, m_t^f)}{\partial \rho_x^{hf}} = \frac{\Gamma_0 \sigma_x^2}{\Gamma_0 \sigma_x^2 + \gamma^2 \sigma_c^2} > 0 \tag{A.1}$$

$$\frac{\partial corr(m_t^h, m_t^f)}{\partial \Psi} = -\frac{2\Gamma_0 \gamma^2 \left(\rho_x^{hf} - \rho_c^{hf}\right) \sigma_x^2 \sigma_c^2}{\psi \left(\Gamma_0 \sigma_x^2 + \gamma^2 \sigma_c^2\right)^2} \tag{A.2}$$

$$\frac{\partial corr(m_t^h, m_t^f)}{\partial \rho_x} = \frac{2\left[\frac{\rho_x}{(1-\rho_x^2)^2} + \frac{\delta^3(1-\gamma\psi)^2}{(1-\rho_x\delta)^3}\right] \gamma^2 (\rho_x^{hf} - \rho_c^{hf}) \sigma_x^2 \sigma_c^2}{\psi^2 \left[\Gamma_0 \sigma_x^2 + \gamma^2 \sigma_c^2\right]^2} \tag{A.3}$$

exist and are well defined. (A.1) is positive for all the values of the parameters that respect the two conditions, implying that the correlation of the two stochastic discount factors is strictly increasing with respect to ρ_x^{hf}. When $\rho_x^{hf} = \rho_c^{hf}$ (A.2) is always zero, meaning that changes in Ψ, γ or δ do not affect the correlation of the two stochastic discount factors. If $\rho_x^{hf} \neq \rho_c^{hf}$, this derivative is zero only if $\psi = \frac{1}{\gamma}\widetilde{\delta}$, where $\widetilde{\delta} = \frac{1-2\rho_x\delta+\delta^2}{\delta^2(1-\rho_x^2)}$. In particular, when $\rho_x^{hf} > \rho_c^{hf}$ the sign of the derivative is positive for $\Psi > \frac{1}{\gamma}\widetilde{\delta}$ and negative for $\psi < \frac{1}{\gamma}\widetilde{\delta}$. Notice that

$$\lim_{\delta \to 1^-} \frac{1}{\gamma}\widetilde{\delta} \geq \frac{1}{\gamma} \quad \text{and} \quad \lim_{\rho_x \to 1^-} \lim_{\delta \to 1^-} \frac{1}{\gamma}\widetilde{\delta} = \frac{1}{\gamma}$$

So, for a high persistence of the long run component and an individual discount factor close to one, the minimum of the cross correlation of the two discount factors is achieved for $\psi = \frac{1}{\gamma}$, that is when the Epstein-Zin preferences collapse to the standard CES utility function. (A.3) is always positive when $\rho_x^{hf} > \rho_c^{hf}$, negative when $\rho_x^{hf} < \rho_c^{hf}$ and equal to zero when $\rho_x^{hf} = \rho_c^{hf}$. Therefore if $\rho_x^{hf} > \rho_c^{hf}$, the minimum is achieved for ρ_x=0. When $\rho_x^{hf} > \rho_c^{hf}$, (A.1), (A.2) and (A.3) imply the existence of one unique minimizer at $\left(\rho_x = 0, \rho_x^{hf} = 0, \psi = \frac{1}{\gamma}\widetilde{\delta}\right)$. $\square$

Proof of Proposition 2. The partial derivative of the variance of the growth of the exchange rate with respect to ρ_x^{hf} exists and is well defined provided that $\rho_x \neq 1$ and $\rho_x \delta \neq 1$:

$$\frac{\partial Var(\pi)}{\partial \rho_x^{hf}} = -2\left[\left(\frac{1}{\psi}\right)^2 \frac{1}{1-\rho_x^2} + \left[\frac{\delta\left(1-\gamma\psi\right)}{\psi\left(1-\rho_x\delta\right)}\right]^2\right]\sigma_x^2 \leq 0 \tag{A.4}$$

In particular, this derivative is always negative, implying that the volatility of the log-depreciation rate achieves its minimum when $\rho_x^{hf} = 1$. □

Appendix B. Unconditional GMM

The GMM in section 3.5 is based on the following 46 unconditional moments:

- $cov\left(\Delta c_t^i, \Delta c_{t-j}^i\right) - (\rho^i)^j \sigma_x^{i2}/(1-\rho^{i2})$, $\forall i \in \{h, f\} \wedge j \in \{0, 1, ..., 4\}$
- $cov\left(\Delta c_t^h, \Delta c_{t-j}^f\right) - (\rho^h)^j \sigma_x^h \sigma_x^f/(1-\rho^h\rho^f)$, $\forall j \in \{0, 1, ..., 4\}$
- $cov\left(\Delta c_t^f, \Delta c_{t-j}^h\right) - (\rho^f)^j \sigma_x^h \sigma_x^f/(1-\rho^h\rho^f)$, $\forall j \in \{1, ..., 4\}$
- $cov\left(\Delta d_t^i, \Delta d_{t-j}^i\right) - (\rho^i)^j (\lambda^i\sigma_x^i)^2/(1-\rho^{i2})$, $\forall i \in \{h, f\} \wedge j \in \{0, 1, ..., 4\}$
- $cov\left(\Delta d_t^h, \Delta d_{t-j}^f\right) - (\rho^h)^j (\lambda^h\sigma_x^h)(\lambda^f\sigma_x^f)/(1-\rho^h\rho^f)$, $\forall j \in \{0, 1, 2\}$
- $cov\left(\Delta d_t^f, \Delta d_{t-j}^h\right) - (\rho^f)^j (\lambda^h\sigma_x^h)(\lambda^f\sigma_x^f)/(1-\rho^h\rho^f)$, $\forall j \in \{0, 1, 2\}$
- $Var\left(\Delta e_{t+1}\right) - \sigma_\pi^2 = 0$
- $E\left[r_{f,t}^i\right] - \overline{r}_f^i$, $Var\left[r_{f,t}^i\right] - \frac{\sigma_x^{i2}}{\Psi^{i2}(1-\rho^{i2})}$ and $cov\left[r_{f,t}^i, r_{f,t-1}^i\right] - \rho^i \frac{\sigma_x^{i2}}{\Psi^{i2}(1-\rho^{i2})}$, $\forall i \in \{h, f\}$
- $E\left[\left(r_{d,t}^i - r_{f,t}^i\right)\right] - \overline{r}_d^{ex,i}$ and $Var\left[\left(r_{d,t}^i - r_{f,t}^i\right)\right] - \Gamma_d^i C^i \Gamma_d^{i\prime}$, $\forall i \in \{h, f\}$
- $cov\left[\left(r_{d,t}^h - r_{f,t}^h\right), \left(r_{d,t}^f - r_{f,t}^f\right)\right] - \rho_{r^{ex}}^{hf}$

where $\overline{r}_f^i$ and $\overline{r}_d^{ex,i}$ are computed in the Technical Appendix by Colacito and Croce (2008b). We implement a standard two-steps GMM procedure. In the first iteration we use a diagonal weighting matrix. Each element of the diagonal is equal to the inverse of the variance of the sample moment we are interested in. In this way our estimate is not unit sensitive. In the second step we minimize the quadratic criterion using the optimal weighting matrix computed at the point estimate of the first step. The statistics we report are obtained using the optimal weighting matrix computed at the final point estimate of the second step. We calibrate μ and μ_d using the sample mean of the consumption and dividends growth both for US and UK.

Appendix C. GMM on Euler Equations restrictions

The predictable components of consumption growth rates are constructed according to either equation (4.4) or (4.5). Innovations to consumption growth and its low frequency components are computed as follows:

$$\begin{aligned} \epsilon^i_{c,t+1} &= \Delta c^i_{t+1} - x^i_t \\ \epsilon^i_{x,t} &= x^i_t - \rho^i x^i_{t-1} \end{aligned}$$

We then use a log-linearized version of the model (equations (2.1)-(2.2)) in order to recover time series of stochastic discount factors and exchange rates. We calibrate the mean of consumption growth to $\mu = .02$ and the subjective discount factor to $\delta = .99$ for both US and UK. The continuous updating GMM procedure of Hansen, Heaton, and Yaron (1996) is applied to the sample counterparts of the following set of Euler equations

- $E\left[M^i_{t+1} R^i_{j,t+1}\right] - 1 = 0, \forall i = \{US, UK\}$ and $\forall j = \{m, f\}$
- $E\left[M^{US}_{t+1} R^{UK}_{j,t+1} \exp^{\Delta e_{t+1}}\right] - 1 = 0, \forall j = \{m, f\}$
- $E\left[M^{UK}_{t+1} R^{US}_{j,t+1} \left(\exp^{\Delta e_{t+1}}\right)^{-1}\right] - 1 = 0, \forall j = \{m, f\}$

and of the following exchange rate moment conditions

- $E\left[M^{US}_{t+1}/M^{UK}_{t+1} - e^{\Delta e_{t+1}}\right] = 0$
- $E\left[\left(M^{US}_{t+1}/M^{UK}_{t+1}\right)^2 - \left(e^{\Delta e_{t+1}}\right)^2\right] = 0$

When preference parameters and coefficients governing the dynamics of the predictable components of consumption growth are jointly estimated, we introduce also the appropriate set of orthogonality conditions.

Appendix D. GMM on the reduced form of the model

The innovations to consumption growth and its predictable component can be computed as in the previous section. The innovation to cash flows is

$$\varepsilon^i_{d,t+1} \quad = \quad \Delta d^i_{t+1} - \lambda^i x^i_t, \forall i \in \{US, UK\}$$

The log-linearized version of the model (equations (2.1)-(2.2)) is then used to construct the sample counterparts of the following moments conditions:

- $E\left[\left(\widetilde{r}^i_{m,t+1} - \widetilde{r}^i_{f,t}\right) - \left(r^i_{m,t+1} - r^i_{f,t+1}\right)\right] = 0, \forall i \in \{US, UK\}$
- $E\left[\widetilde{r}^i_{f,t} - r^i_{f,t+1}\right] = 0, \forall i \in \{US, UK\}$
- $E\left[M^{US}_{t+1}/M^{UK}_{t+1} - \exp\{\Delta e_{t+1}\}\right] = 0$
- $E\left[\left(M^{US}_{t+1}/M^{UK}_{t+1}\right)^2 - \left(\exp\{\Delta e_{t+1}\}\right)^2\right] = 0$

Where $\widetilde{r}^i_{m,t+1}$ and $\widetilde{r}^i_{f,t}$ are the model predictions for the market returns and the risk free rate. When preference parameters and the coefficients governing the dynamics of the predictable components of consumption growth are jointly estimated, we introduce also the appropriate set of orthogonality conditions.

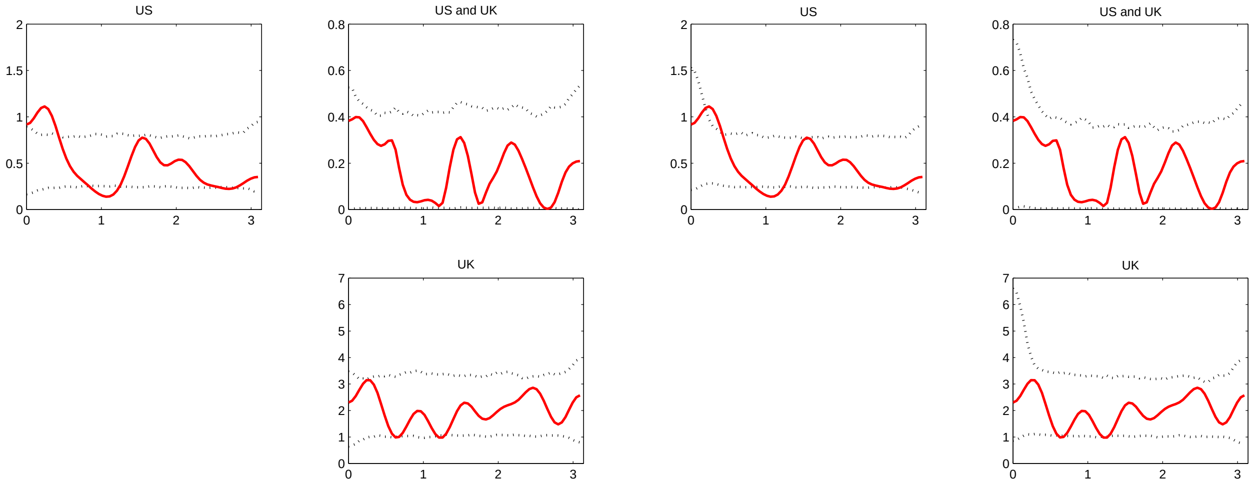

FIG. 1 - Sample periodogram. The solid line is the data sample periodogram, while the dashed lines represent the 95% confidence interval obtained by simulations. In the three panels on the left, consumption growth was simulated as a pure *i.i.d.* process, while in the three panels on the right consumption was simulated as containing a small predictable component calibrated as in the earlier sections.

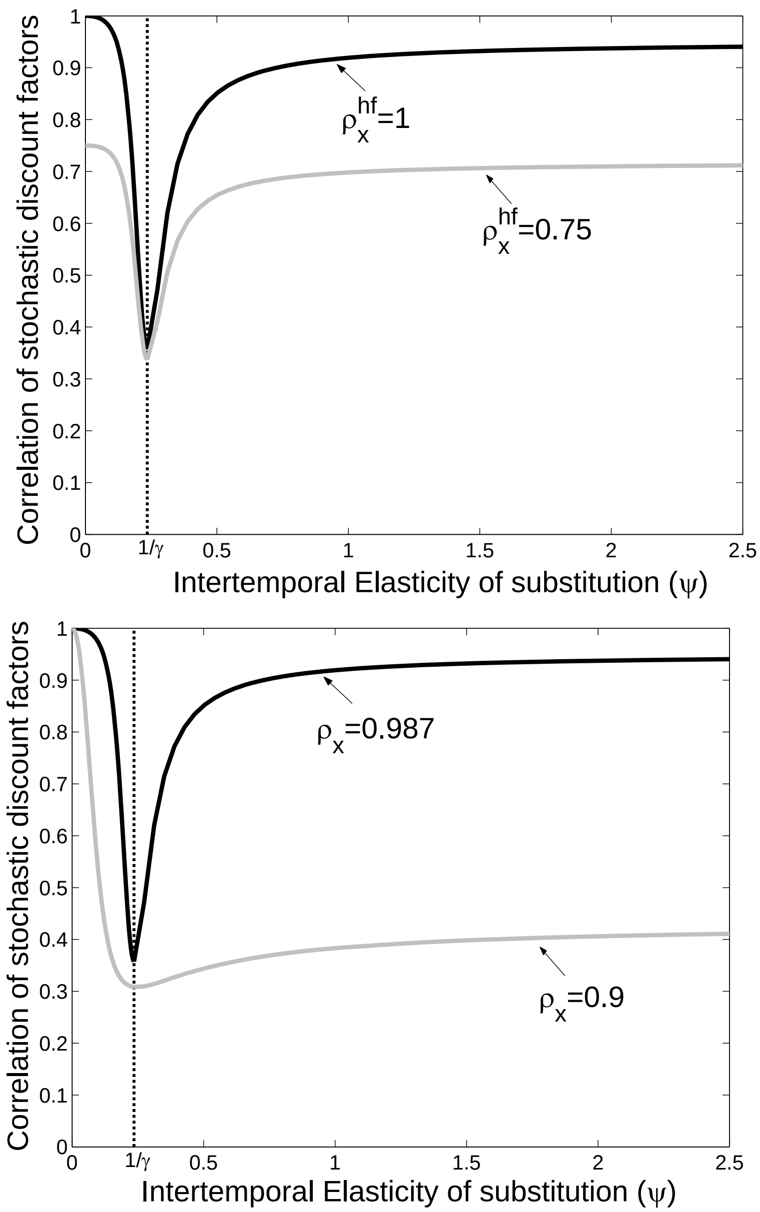

FIG. 2 - The role of intertemporal elasticity of substitution. In both panels, the dark line reports the correlation of stochastic discount factors when ψ changes. The grey line on the top panel is drawn for a smaller value of ρ_x^{hf}, everything else held equal; the grey line on the bottom panel is drawn for a lower ρ_x.

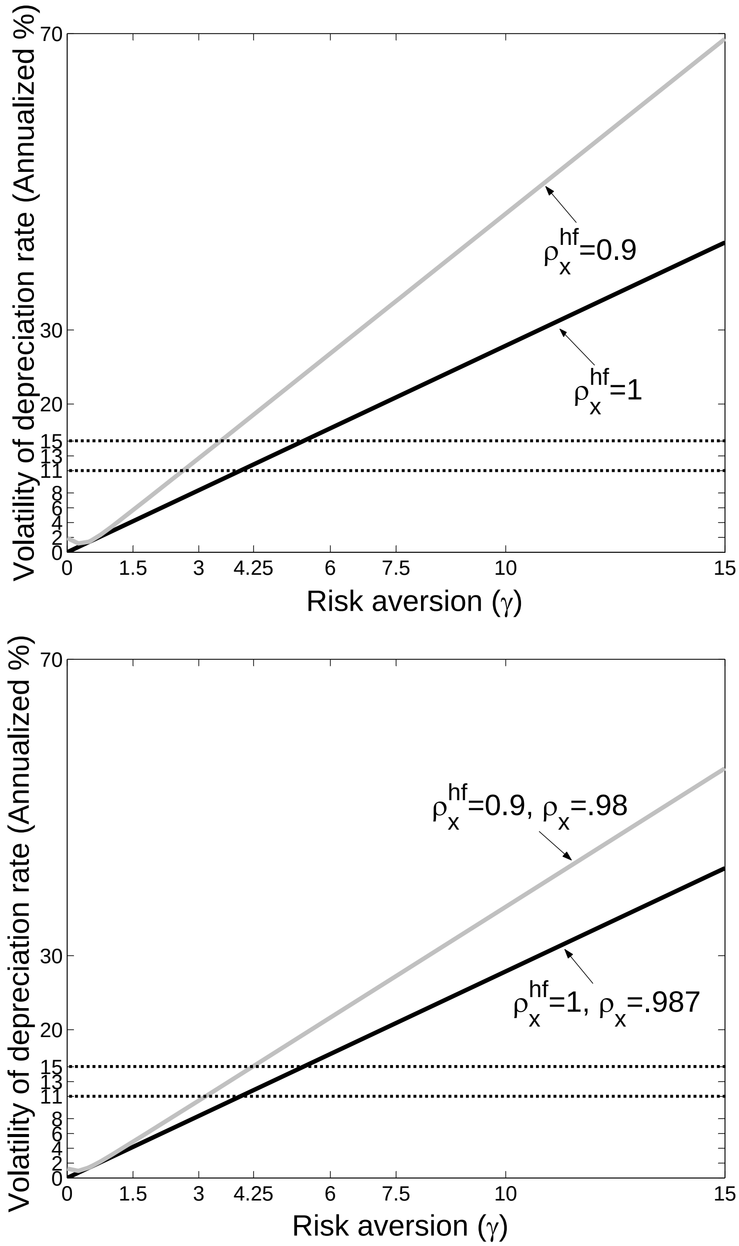

FIG. 3 - The role of risk aversion. In both panels, the dark line reports the volatility of the depreciation rate when γ changes. The grey line in the top panel is drawn for a smaller value of ρ_x^{hf}, everything else held equal; the grey line in the bottom panel is drawn for lower ρ_x and ρ_x^{hf}.

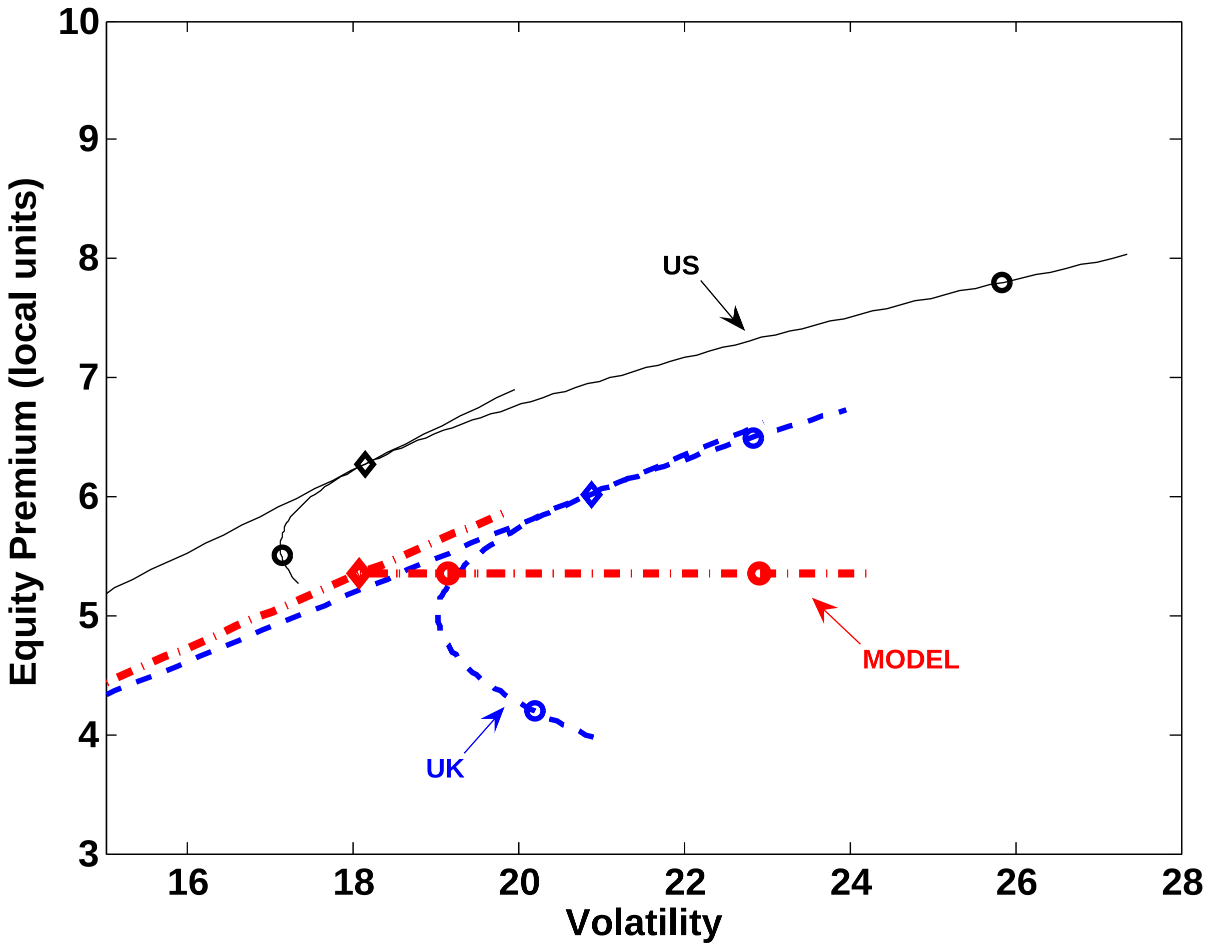

FIG. 4 - The international mean-variance frontier. We plot the investment opportunity sets and the capital allocation lines of a US-based investor (solid curve and line), of a UK-based investor (dashed curve and line), and the related predictions from the model (dash-dot lines). These three curves are drawn under the assumption that each investor has access to the stock market and the risk-free rate of his own country and to the same two assets for the foreign country. The curves for the US and the UK are constructed using quarterly data from 1971:01 to 1998:04. Excess returns are in log units. The model is calibrated as in table 1.

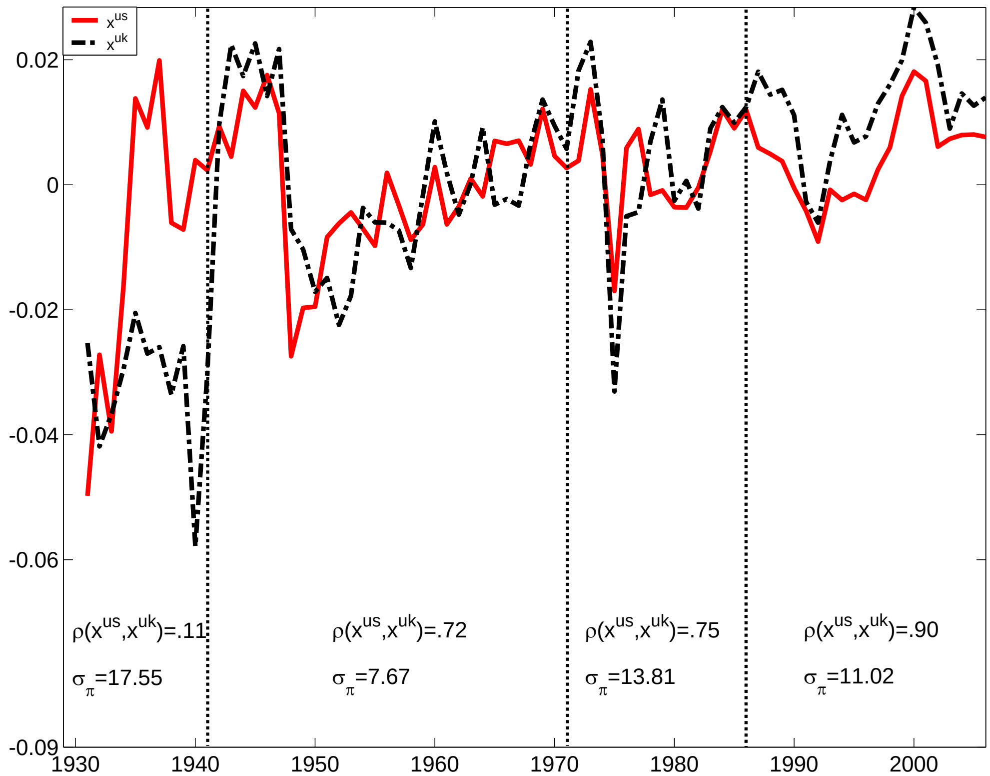

FIG. 5 - Time series of long-run risks in the US and in the UK. The solid line is the projection of US consumption growth on lagged US consumption growth, price-dividend ratio, consumption-output ratio, and default premium. The dash-dot line is the projection of UK consumption growth on lagged UK consumption growth, price-dividend ratio, and consumption-output ratio. The numbers between the vertical dotted lines report the correlation of these projections and the volatility of the growth of the exchange rate on the corresponding subsample. Data are annual (1929–2006). The OLS–estimates of the projections are reported in table 4.

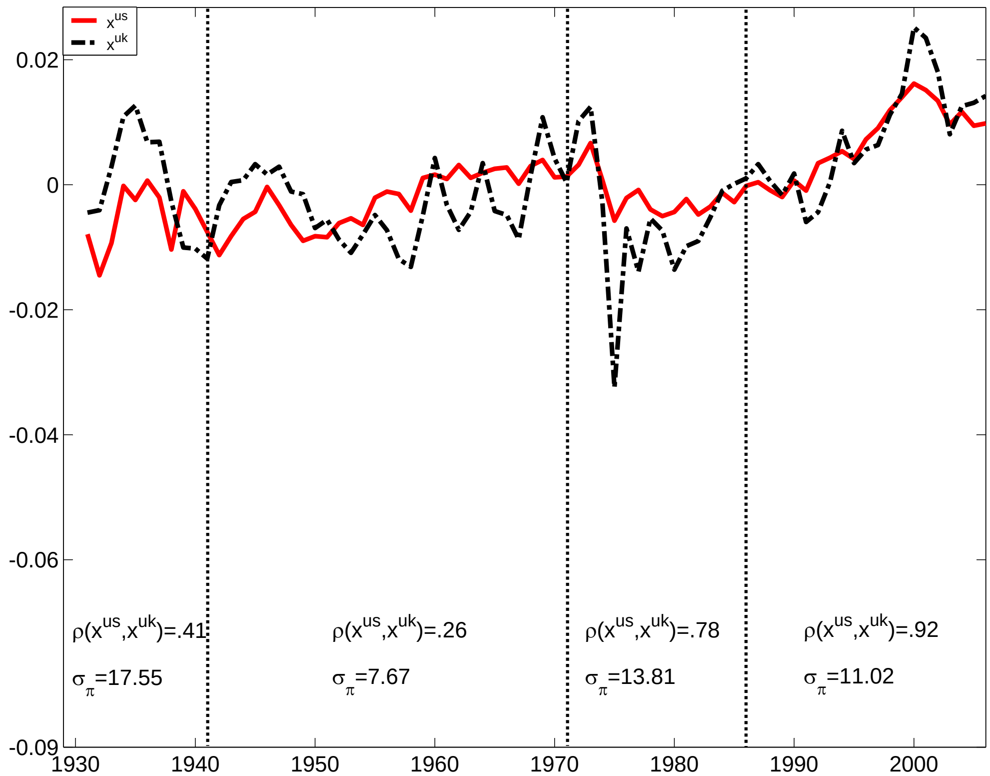

FIG. 6 - Time series of long-run risks in the US and in the UK. The solid line is the projection of US consumption growth on lagged US price-dividend ratio. The dash-dot line is the projection of UK consumption growth on lagged UK price-dividend ratio. The numbers between the vertical dotted lines report the correlation of these projections and the volatility of the growth of the exchange rate on the corresponding sub-sample. Data are annual (1929–2006). The OLS–estimates of the projections are reported in table 4.

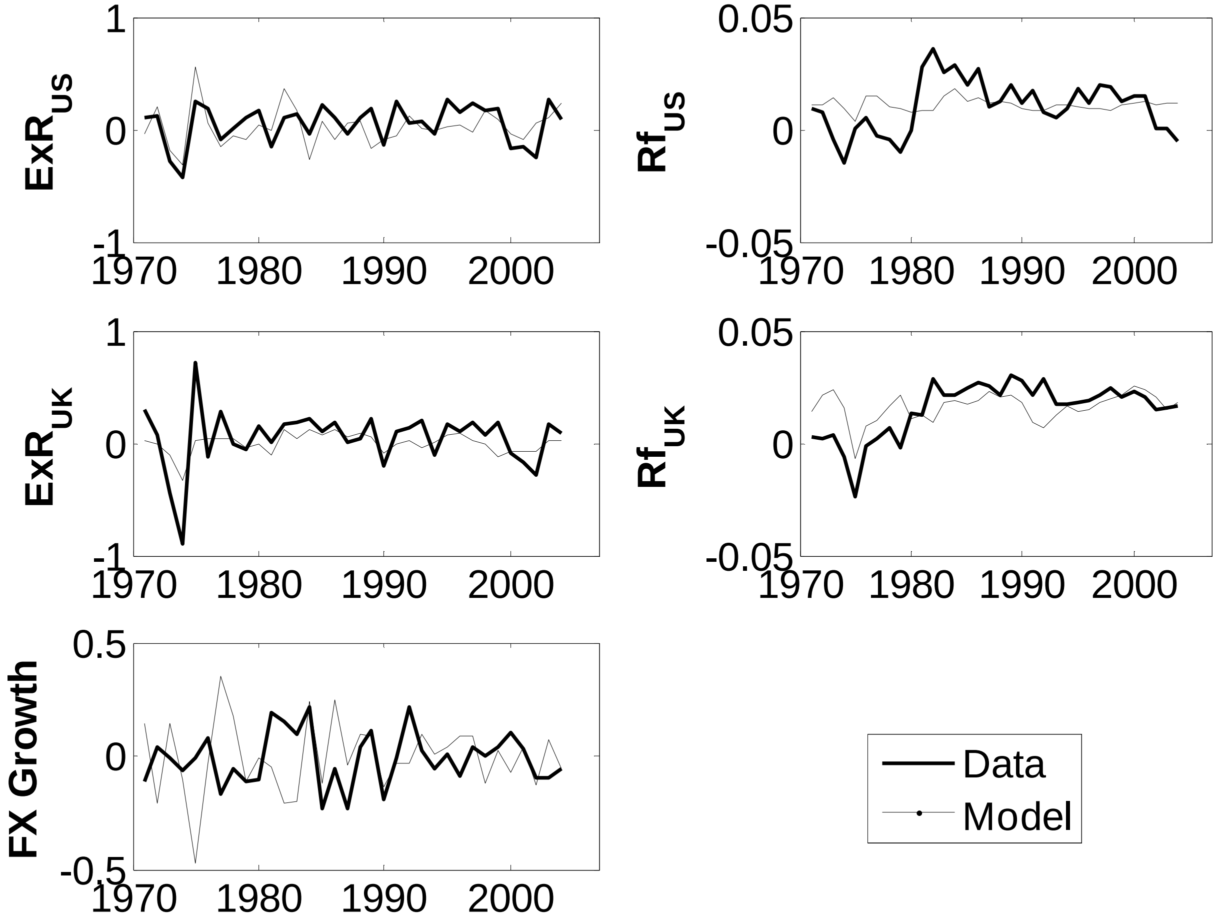

FIG. 7 - Predictions from continuous GMM applied to annual consumption and dividends (1971–2006). In each panel the solid thick line refers to the data, while the thin line shows the prediction, at the point estimate, of the model with long-run risk in both consumption and dividends. The long-run component is estimated using all the predictive variables; all the parameters are calibrated as in table 8, last column. 'ExR' stands for excess return, 'Rf' for risk-free rate. The bottom panel shows the exchange rate growth. All variables are in log units.

TABLE 1

BASELINE CALIBRATION.

Ψ	Intertemporal elasticity of substitution	2
γ	Risk aversion	4.25
δ	Subjective discount factor	0.998
μ_c	Average consumption growth	15×10^{-4}
ρ	Autoregressive coefficient of the long-run component x_t	0.987
φ_e	Ratio of long-run shock and short-run shock volatilities	0.048
σ	Standard deviation of the short-run shock to consumption	68×10^{-4}
μ_d	Average dividend growth	.0007
λ	Leverage	3.0
φ_d	Volatility ratio of short-run shocks to dividend and consumption growth	5.0
ρ_x^{hf}	Cross-country correlation of the long-run shock	1.0
ρ_c^{hf}	Cross-country correlation of the short-run shock to consumption	0.3
ρ_d^{hf}	Cross-country correlation of the short-run shock to dividends	-0.1

Notes - The countries share the same calibration. The model describes a monthly decision problem, and as a consequence we set the subjective discount factor to 0.998. The intertemporal elasticity of substitution, ψ, is equal to 2 as estimated by Bansal, Gallant, and Tauchen (2007). The parameters of the laws of motion of consumption growth are set to reproduce the average behavior of consumption growth in the United States and in the United Kingdom. We set $\rho_x = .987$, which is the value estimated by Bansal, Gallant, and Tauchen (2007). The standard deviation of consumption growth implied by our choice of parameters is approximately 2.4% in annualized terms, which is in-between the average growth of per capita consumption of nondurables and services from 1970 to 1998 for the US and the UK. The cross-country correlations of the idiosyncratic shocks to consumption and shocks to the trend are chosen as to obtain a correlation of consumption growths in the order of 0.3. The coefficient λ is set in such a way that the ratio $\sigma_{\Delta d}/\sigma_{\Delta c}$ is equal to 4.86 that is in the range $(4, 8)$ estimated by Ludvigson, Lettau, and Wachter (2004). The cross-country correlation of the short-run shocks to dividends, ρ_d^{hf}, is set so as to achieve an almost zero correlation of dividend growths between the US and the UK.

TABLE 2
KEY MOMENTS OF INTERNATIONAL MARKETS - SYMMETRIC CALIBRATION

		US	UK	Model
$\rho\left(m^h, m^f\right)$	Correlation of SDF	-	-	0.931
$E(\Delta e)$	Average depreciation rate	1.330		0.000
$\sigma(\Delta e)$	Volatility of depreciation rate	11.211		11.832
$E(r_d - r_f)$	Average excess return, local units	5.504	6.501	5.346
$\sigma(r_d - r_f)$	Volatility of excess return	17.130	22.830	19.132
$\rho\left(r_d^h - r_f^h, r_d^f - r_f^f\right)$	Correlation of excess returns, local units	0.670		0.603
$E(r_f)$	Average risk free rate	1.470	1.620	1.332
$\sigma(r_f)$	Volatility of risk free rate	1.530	2.920	1.191
$\rho\left(r_f^h, r_f^f\right)$	Correlation of risk free rates	0.653		1.000
$\frac{E(r_d - r_f + \Delta e)}{\sigma(r_d - r_f + \Delta e)}$	Foreign Sharpe-Ratio in domestic units	0.200	0.312	0.227
$\rho\left(r_d^h - r_f^h, r_d^f - r_f^f + \Delta e\right)$	Correlation of excess returns, domestic units	0.631	0.600	0.543
$\rho(r_d - r_f, \Delta e)$	Correlation of excess returns, local units, and depr. rate	0.040	-0.030	0.000
$\sigma(\Delta c)$	Volatility of consumption growth	1.369	2.860	2.450
$\frac{\sigma^2(x)}{\sigma^2(\Delta c)} \times 100$	Share of predictable cons. variance	-	-	8.180
$\sigma(\Delta d)$	Volatility of dividend growth	16.851	6.872	11.961
$\rho\left(\Delta c^h, \Delta c^f\right)$	Correlation of consumption growth	0.280		0.351
$\rho\left(\Delta d^h, \Delta d^f\right)$	Correlation of dividend growth	-0.03		-0.07

Notes - Data are quarterly, real per-capita and from 1971:01 to 1998:04. Means and variances are annualized and multiplied by 100. For the model all coefficients are set to the numbers reported in Table 1. All variables are in log-units. We simulate the model at monthly frequency and time-aggregate the data to an annual frequency. 'Local units' stands for variables expressed in terms of local real consumption. 'Domestic (foreign) units' stands for variables measured in terms of the same real domestic (foreign) consumption good.

TABLE 3
PRICES AND LONG RUN RISKS.

	Consumption only		Whole Model	
	Point Estimate	Std Err	Point Estimate	Std Err
ρ^h	0.912	(0.121)	0.996	(0.003)
ρ^f	0.970	(0.053)	0.961	(0.026)
σ_x^h	0.131	(0.109)	0.013	(0.007)
σ_x^f	0.126	(0.112)	0.118	(0.042)
σ^h	0.546	(0.055)	0.296	(0.066)
σ^f	1.184	(0.112)	0.551	(0.122)
ρ_c^{hf}	0.219	(0.106)	0.204	(0.343)
ψ^h	-	-	1.842	(0.222)
ψ^f	-	-	1.534	(0.276)
γ^h	-	-	9.000	(3.776)
γ^f	-	-	7.692	(3.535)
λ^h	-	-	8.990	(4.055)
λ^f	-	-	3.179	(0.493)
σ_d^h	-	-	7.073	(0.263)
σ_d^f	-	-	1.564	(0.110)
ρ_d^{hf}	-	-	-0.114	(0.057)

		Unconditional moments (at point estimates)	
		US	UK
$E\left(\frac{e_{t+1}}{e_t}\right)$	-	1.234	
$\sigma\left(\frac{e_{t+1}}{e_t}\right)$	-	10.787	
$\rho\left(m^h, m^f\right)$	-	0.972	
$\rho\left(r_f^h, r_f^f\right)$	-	0.570	
$\rho\left(r_d^h - r_f^h, r_d^f - r_f^f\right)$	-	0.670	
$\rho\left(r_d^h - r_f^h, r_d^f - r_f^f + \Delta e\right)$	-	0.601	0.530
$\frac{E(r_d - r_f + \Delta e)}{\sigma(r_d - r_f + \Delta e)}$	-	0.163	0.291

Notes - We implement a standard two-steps GMM estimation as described in Appendix B on quarterly data (1971:01-1998:04). The second and third columns report the results of the estimation on consumption data only, while the fourth and fifth columns report the results of the estimation of the whole model. In parenthesis we report the standard errors multiplied by 100. The bottom panel of the table reports annualized unconditional moments measured at the point estimates. The subjective discount factors δ^h and δ^f are equal to the numbers reported in Table 1 raised to the third power to account for the quarterly frequency of the model. We calibrate μ and μ_d using the sample mean of the consumption and dividends growth both for US and UK. The following relabeling is used to simplify the estimation routine: $\sigma_x^i = \varphi_e^i \sigma^i$ and $\sigma_d^i = \varphi_d^i \sigma^i$.

TABLE 4
PREDICTIVE REGRESSIONS ON CONSUMPTION

	Δc_{t-1}	pd_{t-1}	cy_{t-1}	$default_{t-1}$	$R^2_{\Delta c}$	ρ_x	R^2_x
US	0.508	0.012	-0.061	1.252	38.321	0.645	49.164
	(0.101)	(0.005)	(0.029)	(0.456)		(0.093)	
	-	0.014	-	-	8.626	0.885	76.049
		(0.008)				(0.065)	
UK	0.207	0.029	-0.100	-	27.759	0.771	60.487
	(0.115)	(0.013)	(0.034)			(0.090)	
	-	0.033	-	-	7.574	0.726	51.264
		(0.012)				(0.099)	

Notes - Columns 2-5 report the estimated parameters of the regressions

$$\Delta c_t = \beta_0 \Delta c_{t-1} + \beta_1 pd_{t-1} + \beta_2 \Delta cy_{t-1} + \beta_3 default_{t-1} + \varepsilon_t^c$$

for US and UK on de-meaned annual data from 1929 to 2006. Column 6 shows the R^2 of these regressions. Column 7 reports the point estimates of the regressions

$$x_t = \rho_x x_{t-1} + \varepsilon_t^x$$

where x_t is the forecast of Δc_t obtained using the above predictive regressions. The last column shows the R^2 of these last regressions. The numbers in parenthesis are Newey-West adjusted standard errors.

TABLE 5
GMM ESTIMATION WITH ALL PREDICTIVE VARIABLES

Sample	31-06	71-06	74-06	31-06	71-06	74-06
ψ_{US}	21.914	4.843	3.108	1.422	1.459	1.459
	(155.989)	(7.661)	(4.002)	(2.284)	(4.693)	(8.286)
γ_{US}	1.924	2.867	4.099	4.057	2.237	2.237
	(0.357)	(0.518)	(0.543)	(17.755)	(11.928)	(16.486)
ψ_{UK}	12.458	0.910	2.484	1.878	0.953	0.953
	(48.231)	(0.405)	(3.709)	(7.967)	(1.856)	(0.775)
γ_{UK}	1.449	2.562	2.422	1.913	8.159	8.159
	(0.233)	(0.354)	(0.412)	(5.257)	(3.547)	(3.997)
β_{US}^{dc}	-	-	-	0.135	0.129	0.399
				(0.088)	(0.129)	(0.132)
β_{US}^{pd}	-	-	-	0.006	0.006	0.006
				(0.004)	(0.004)	(0.004)
β_{US}^{cy}	-	-	-	0.018	0.028	0.028
				(0.041)	(0.142)	(0.129)
β_{US}^{def}	-	-	-	0.462	1.216	1.216
				(0.479)	(0.396)	(0.364)
β_{UK}^{dc}	-	-	-	0.283	0.181	0.181
				(0.067)	(0.068)	(0.091)
β_{UK}^{pd}	-	-	-	0.012	0.009	0.009
				(0.008)	(0.012)	(0.011)
β_{UK}^{cy}	-	-	-	-0.017	0.027	0.027
				(0.027)	(0.027)	(0.099)
ρ_x^{US}	-	-	-	0.989	0.999	0.999
				(0.175)	(0.267)	(0.253)
ρ_x^{UK}	-	-	-	0.999	0.821	0.821
				(0.26)	(0.172)	(0.154)
J-stat	32.013	32.574	32.629	33.47	19.962	16.138
	(0.000)	(0.000)	(0.000)	(0.000)	(0.003)	(0.013)
σ_π	10.237	7.527	9.059	57.209	41.855	57.221
ρ_{m^h,m^f}	0.269	0.664	0.702	0.305	0.429	0.517

Notes - Each column reports the parameters estimated using continuous GMM on a different sample and on a different set of moments conditions. The sample is specified in the first row of each column. Data are annual. All GMMs have the following set of moments conditions in common: Euler equations for domestic stock market returns and risk free rates (4), Euler equations for foreign stock market returns and risk free rates expressed in local units (4), and first two moments of the exchange rates growth (2). The GMM in the first three columns are conditional on the OLS estimates of the regression of condumption growth on the price-dividend ratio of each country, as documented in table 4. For the results reported in the last three columns all models' parameters are estimated jointly by adding the appropriate set of orthogonality restrictions. The numbers is parenthesis below each estimate are Newey-West adjusted standard errors. The numbers in parenthesis below each J-stat are p-values. The last two rows of each column report the implied correlation of stochastic discount factors and volatility of exchange rate growth at the point estimates.

TABLE 6

GMM ESTIMATION WHEN PRICE DIVIDEND RATIO IS ONLY PREDICTOR

Sample	31-06	71-06	74-06	31-06	71-06	74-06
ψ^{US}	1.480	0.241	0.253	1.014	2.828	2.076
	(1.156)	(0.113)	(0.111)	(0.7935)	(4.815)	(4.631)
γ^{US}	4.703	20.087	19.923	7.722	5.739	7.92
	(0.842)	(2.059)	(2.322)	(1.7538)	(9.311)	(21.874)
ψ^{UK}	0.824	1.821	3.665	1.21	0.949	2.383
	(0.555)	(3.968)	(16.188)	(1.4469)	(0.434)	(3.812)
γ^{UK}	4.545	6.405	6.311	1.398	2.99	2.054
	(0.601)	(0.848)	(0.814)	(3.3732)	(1.937)	(1.396)
β_{pd}^{US}	-	-	-	0.002	0.007	0.003
				(0.0007)	(0.004)	(0.003)
β_{pd}^{UK}	-	-	-	0.036	0.034	0.016
				(0.0114)	(0.007)	(0.004)
ρ_x^{US}	-	-	-	0.986	0.977	0.990
				(0.0179)	(0.061)	(0.056)
ρ_x^{UK}	-	-	-	0.998	0.933	0.998
				(0.167)	(0.086)	(0.038)
J-stat	30.531	24.652	22.013	27.009	16.622	11.589
	(0.000)	(0.002)	(0.005)	(0.000)	(0.011)	(0.072)
σ_π	18.096	24.365	23.585	25.609	19.757	26.734
ρ_{m^h,m^f}	0.361	0.692	0.696	0.348	0.718	0.700

Notes - Each column reports the parameters estimated using continuous GMM on a different sample and on a different set of moments conditions. The sample is specified in the first row of each column. Data are annual. All GMMs have the following set of moments conditions in common: Euler equations for domestic stock market returns and risk free rates (4), Euler equations for foreign stock market returns and risk free rates expressed in local units (4), and first two moments of the exchange rates growth (2). The GMM in the first three columns are conditional on the OLS estimates of the regression of condumption growth on the price-dividend ratio of each country, as documented in table 4. For the results reported in the last three columns all models' parameters are estimated jointly by adding the appropriate set of orthogonality restrictions. The numbers is parenthesis below each estimate are Newey-West adjusted standard errors. The numbers in parenthesis below each J-stat are p-values. The last two rows of each column report the implied correlation of stochastic discount factors and volatility of exchange rate growth at the point estimates.

TABLE 7
PREDICTIVE REGRESSIONS ON DIVIDENDS

	Δc_{t-1}	pd_{t-1}	cy_{t-1}	$default_{t-1}$	λ	$R^2_{\Delta d}$	LR-test
US	0.470	0.010	-0.065	1.291	2.485	0.061	7.998
	(0.136)	(0.006)	(0.042)	(0.733)	(1.075)		(.046)
	-	0.014	-	-	5.965	0.113	-
		(0.006)			(2.252)		
UK	0.229	0.030	-0.090	-	1.597	0.119	1.680
	(0.087)	(0.020)	(0.030)		(0.571)		(.431)
	-	0.033	-	-	2.297	0.067	-
		(0.024)			(1.360)		

Notes - In Columns 2-6, we report the maximum likelihood estimates of $\left\{\beta_j^i\right\}_{j=0}^3$ of the following restricted system of equations:

$$
\begin{aligned}
\Delta c_t^i &= \beta_0^i \Delta c_{t-1}^i + \beta_1^i pd_{t-1}^i + \beta_2^i \Delta cy_{t-1}^i + \beta_3^i default_{t-1}^i + \varepsilon_{c,t}^i \\
\Delta d_t^i &= \lambda \left(\beta_0^i \Delta c_{t-1}^i + \beta_1^i pd_{t-1}^i + \beta_2^i \Delta cy_{t-1}^i + \beta_3^i default_{t-1}^i\right) + \varepsilon_{d,t}^i
\end{aligned}
$$

for all countries $i = \{US, UK\}$. Standard errors are reported in parenthesis. The next column reports the estimates of λ^i. The seventh column shows the R^2 of these regressions. The unrestricted version of the model is:

$$
\begin{aligned}
\Delta c_t^i &= \beta_0^i \Delta c_{t-1}^i + \beta_1^i pd_{t-1}^i + \beta_2^i \Delta cy_{t-1}^i + \beta_3^i default_{t-1}^i + \varepsilon_{c,t}^i \\
\Delta d_t^i &= \phi_0^i \Delta c_{t-1}^i + \phi_1^i pd_{t-1}^i + \phi_2^i \Delta cy_{t-1}^i + \phi_3^i default_{t-1}^i + \varepsilon_{d,t}^i
\end{aligned}
$$

The last column reports the Likelihood Ratio tests with the associated p-values in parenthesis. Data are annual, demeaned. The sample ranges from 1929 to 2006.

TABLE 8
GMM ESTIMATION WITH DIVIDENDS

Sample	31-06	71-06	31-06	71-06
ψ_{US}	2.880	4.092	2.172	2.895
	(9.707)	(6.637)	(9.990)	(.767)
γ_{US}	2.077	7.640	2.58	2.873
	(9.011)	(8.059)	(12.451)	(1.950)
ψ_{UK}	1.945	1.226	1.599	1.746
	(12.792)	(0.192)	(11.462)	(1.655)
γ_{UK}	4.117	1.769	2.969	1.294
	(20.698)	(0.641)	(55.955)	(.413)
λ_{US}	3.297	9.432	9.517	2.787
	(8.025)	(4.681)	(18.013)	(.804)
β_{US}^{dc}	-	-	0.001	0.571
			(.127)	(.142)
β_{US}^{pd}	0.010	0.005	0.004	0.013
	(0.004)	(.003)	(.002)	(.005)
β_{US}^{cy}	-	-	-0.001	0.468
			(.030)	(.153)
β_{US}^{def}	-	-	0.000	1.116
			(.480)	(.414)
λ_{UK}	8.025	1.167	6.814	0.987
	(3.052)	(1.234)	(19.859)	(1.242)
β_{UK}^{dc}		-	-.000	0.257
			(.116)	(.211)
β_{UK}^{pd}	0.007	0.025	0.001	0.023
	(.005)	(0.006)	(.015)	(0.013)
β_{UK}^{cy}	-	-	0.005	-0.076
			(.010)	(0.109)
ρ_x^{US}	0.982	0.976	0.999	0.858
	(.113)	(0.023)	(.081)	(.134)
ρ_x^{UK}	0.972	0.992	0.997	0.932
	(.312)	(0.023)	(.529)	(.148)
J-stat	16.411	6.67	34.387	5.278
	(.000)	(0.035)	(.000)	(.259)
σ_π	0.235	0.242	0.286	0.164
ρ_{m^h,m^f}	0.432	0.716	0.364	0.59

Notes - Each column reports the parameters estimated using GMM with continuous updating on a different sample and on a different set of moments conditions. Data are annual. The sample is specified in the first row of each column. All GMMs have the following set of moment conditions in common: mean of domestic log excess returns and risk free rates (4); first two centered moments of the exchange rates growth (2). All models' parameters are estimated jointly by adding the appropriate set of orthogonality restrictions. The numbers in parenthesis below each estimate are Newey-West adjusted standard errors. The numbers in parenthesis below each J-stat are p-values. The last two rows of each column report the implied correlation of stochastic discount factors and volatility of exchange rate growth at the point estimates.