COINTEGRATION RANK TEST AND LONG RUN SPECIFICATION: A NOTE ON THE ROBUSTNESS OF STRUCTURAL DEMAND SYSTEMS

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Abstract

Private demand systems provide a practical application for analyzing identification issues in cointegration analysis. The paper conducts Montecarlo simulation experiments of cointegrated demand systems by assuming non-separability of government consumption. This framework enables further to test the robustness of models under alternative empirical specifications in which the homogeneity restriction is assumed to hold. The results highlight that separability of utility function with respect to government spending and the over-inclusion of lagged dependent variables introduce important bias in identifying the long run demand system, while the model specification with homogeneity restriction perform better when the theoretical hypothesis is contained in the data.

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1. INTRODUCTION

The theory-identification approach in modelling dynamic relationships of long run economic "facts" is currently a crucial step for the estimation and testing strategy of private demand systems representing a robust framework to analyze consumption decisions and welfare (Pesaran and Shin, 2002). Given their general properties, demand systems have been required to answer even more specific policy questions. For example, in analyzing the impact of fiscal policies on private consumption decisions, these models have been extended to remove the strict separable assumption with respect to government consumption (Tridimas, 2002; Aristei and Pieroni, 2008).

The methodological contribution of the paper is to verify how the inclusion of government spending variables affects the identification properties of dynamic demand systems. Specifically, under the theoretical hypothesis that government spending variables are weakly exogenous with respect to private consumption decisions and I(1), we document the results of the simulated rank of the Johansen cointegration trace test in limited sample. We use the exact-identified conditional long run demand system as a benchmark to compare power and size properties of the cointegrating rank test with respect to the separable demand model. The second contribution is to show how imposing the homogeneity constraint in the model at an early stage can help to improve the statistical performance of the identification tests, reducing the spurious rejection rate when this theoretical property is not rejected by the sample of data.

To achieve these aims, we propose Monte Carlo simulations by using estimated non-separable reference model as the data generating process (DGP). The system is adapted to be similar to the type commonly estimated in the demand system literature and considers as a typical sample size the UK quarterly data (1964Q1-2002Q4) used in Aristei and Pieroni (2008).

The rest of the paper is organized as follows. Section II outlines the empirical identifying framework based on the extension of private demand systems to include publicly provided goods. In section III, the simulation experiments assess the identifying conditions under different nested specifications and the robustness of the findings by means of power and size properties. Section IV concludes the paper.

2. IDENTIFYING LONG RUN DEMAND SYSTEMS

We present the econometric framework of a non-separable demand system and describe the procedures used to identify the models in line with theory suggestions. Then, by incorporating separability with respect to government consumption or by imposing theoretical properties of homogeneity at an early stage, nested models are selected and statistical properties of the cointegrating rank test evaluated and compared.

We begin by specifying a vector autoregressive (VAR(p)) for y_t as a structural vector error correction model (VECM),

$$A_0 \Delta y_t = A_1 \Delta y_{t-1} + ... + A_p \Delta y_{t-p} + A^* y_{t-p} + \xi_t \qquad [1]$$

where A_i ($i = 1, 2, ..., p$) is a $n \times n$ matrix of unknown short-run parameters and A^* is the correspondent matrix of the long-run parameters. The reduced form is obtained by multiplying the equation [1] by the inverse of contemporaneous coefficient matrix A_0^{-1}. Formally:

$$\Delta y_t = \Gamma_1 \Delta y_{t-1} + ... + \Gamma_p \Delta y_{t-p} + \Pi y_{t-p} + \varepsilon_t \qquad [2]$$

where constant terms are included in the matrix Π, while ε_t represents the vector of disturbances. Though in principle we are interested to a complete identification of short and long-run parameters, an incomplete cointegrating rank of the long-run matrix makes this aim ineffective. We follow the long-run identification procedure formulated by Pesaran and Shin (2002) that enables to evaluate alternative models, to test theory and to overcome the econometric issues arisen in the work of Lewbel and Ng (2005).

Before revising this identification strategy, that we will apply to long run demand system, we generalize model [2] to account for the effects of non-separable exogenous I(1) variables. We rewrite the model by the conditional and marginal equations under the assumption that the process $\{z_t\}_{t=1}^{\infty}$ is weakly exogenous with respect to Π and this stochastic process is assumed as long-run forcing for y_t (Granger and Lin,1995):

$$\Delta y_t = \gamma_0 + \Psi \Delta z_t + \sum_{i=1}^{p-1} \Phi_i \Delta x_{t-i} + \Pi_y x_{t-1} + \varepsilon_{yt} \qquad [3]$$

$$\Delta z_t = b_{z0} + \sum_{i=1}^{p-1} \Gamma_{zi} \Delta x_{t-i} + u_t \qquad [4]$$

where the vector x_t (with dimension $m = n + k$) is partitioned into a $n \times 1$ vector of endogeneous variables y_t and a $k \times 1$ vector of exogeneous variables z_t, that represents the long-run I(1) forcing variables for y_t and Π_y is the matrix of coefficients of the conditional long run model. It is worth noting that the error term ε_t (and its covariance matrix) is partitioned conformably to x_t as $\varepsilon_t = (\varepsilon'_{yt}, \varepsilon'_{zt})'$, expressing ε_{yt} conditionally on ε_{zt} as:

$$\varepsilon_{yt} = \Lambda_{yz} \Lambda^{-1}{}_{zz} \varepsilon_{zt} + u_t \qquad [5]$$

where the innovations u_t are distributed as $N(0, \Lambda_{uu})$ and $\Lambda_{uu} \equiv \Lambda_{yy} - \Lambda_{yz} \Lambda^{-1}{}_{zz} \Lambda_{zy}$ are independent of ε_{zt}.

Let the conditional model [3], the reduced rank *p*th vector autoregressive (VAR) model is firstly re-written for estimating and testing cointegrating vectors β in the matrix Π_y:

$$\Delta y_t = \gamma_0 + \Psi \Delta z_t + \sum_{i=1}^{p-1} \Phi \Delta x_{t-i} + \alpha_y \beta' x_{t-1} + \varepsilon_{yt} \qquad [6]$$

where the $\alpha_y \beta$ matrix is decomposed in the $n \times r$ loadings matrix α_y and the $(n+k) \times r$ matrix of cointegrating vectors β that are full column rank and identified by a non-singular matrix $(r \times r)$. The model in this form is denoted as $H^*(r)$. A test for the rank hypothesis of the model [6] consists in the rejection of small values of the likelihood ratio $L(H^*(r))/L(H^*(n))$ adapted for the case in which $k>0$ (Pesaran et al., 2000). Under the Gaussian distribution of errors, the test statistic is known as the trace statistic Q_r^*, (Johansen, 1995). Formally,

$$Q_r^* = -2Log\left(\frac{L(H^*(r))}{L(H^*(n))}\right) = (T-K)\sum_{i=r+1}^{m}\log(1-\hat{\lambda}_i) \qquad [7]$$

where $\hat{\lambda}_1 > .... > \hat{\lambda}_n > \hat{\lambda}_{n+1} = 0$ are the ordered roots of the $\det[\lambda S_{11} - S_{10}S_{00}^{-1} \times S_{01}] = 0$. The matrices $S_{ij}, i, j = 0,1$ are defined by $S_{ij} = (T-p)^{-1}\sum_{t=p+1}^{T} R_{it}R_{jt}^{'}$ in which R_{0t} and R_{1t} are the residuals of the differences ΔY_t and the variable incorporated in x_{t-1} after regressing on $\Delta Y_{t-1}, ..., \Delta Y_{t-p+1}$ and 1. The extension of the standard Johansen asymptotic distribution of rank tests for models with exogenous non-stationary forcing variables and the statistical distribution of the trace statistic are reported in Harbo et al. (1998) and Pesaran et al. (2000).

This is a useful approach to derive a conditional long run demand system that is in line with the predictions of consumer behaviour theory under quantity constraints (Pollak, 1969, 1971; Neary and Roberts, 1980; Deaton, 1981)[1], that allows testing for the identifying conditions and for the assumption of separability between the utility offers by government consumption in private expenditure. As a convenient dynamic framework, we re-parameterize the static version of the Almost Ideal Demand System (AI) developed by Deaton and Muellbauer (1980), in which expenditure equations are extended by including government consumption, as in Aristei and Pieroni (2008). We begin from the baseline formulation of the model in budget shares (*w*),

$$w_i = \alpha_i + \sum_{k=1}^{n}\gamma_{ik}\ln p_k + \mu_i\left[\ln e - \ln \boldsymbol{P}\right] + \sum_{j=1}^{m}\theta_{ij}g_j \qquad i,k = 1,2,...,n \quad j = 1,2,...,m \qquad [8]$$

[1] Differently from Fleissig and Rossana (2003), we realistically assume that the public provision of goods and services is not freely chosen but established a priori by the government decisions.

where p_k is the relative price of the *k-th* good, e represents total per capita expenditure, $\ln \boldsymbol{P} = \alpha_0 + \sum_{i=1}^{n} (\alpha_i + \sum_{j=1}^{m} \theta_{ij} g_j) \ln p_i + (1/2) \sum_{i=1}^{n} \sum_{j=1}^{m} \gamma_{ik} \ln p_i \ln p_k$ is a translog price index, g_j represents real government expenditures. γ_{ik}, μ_i, θ_{ij} are the associated parameters, while α_i are the constant parameters. We extend the static model to obtain the equivalent specification of the equation [6] for the conditional long run demand system in which a crucial role for the identification (and estimation) is carried out by the non-testable *adding-up* theoretical constraint. This implies the specification of a reduced rank VAR model for the $n-1$ equations of the demand system. To exactly recover long run structural parameters of the model, theory suggests that the number of cointegrating relationships r has to be equal to the number of non-singular demand equations, $n-1$, expressed in budget shares. If the matrix of cointegration vectors $\beta^{'}$ for demand system is specified as:

$$\beta^{'} = (\beta *^{'}, \theta) = [W_{n-1}, -B, -\theta] \qquad [9]$$

where W_{n-1} is the $n-1 \times n-1$ budget share matrix that serves for imposing long run identifying conditions, $-B$ the matrix of the parameters for log prices and real income and $-\theta = -(\theta_{1j}, ..., \theta_{n-1j})$ the parameters for the effect of government consumption on private categories of consumption, the explicit r cointegrating vectors for the demand system is given as,

$$cv(r) = \beta' x_{t-1} = \beta'(w_{1t-1}, ..., w_{n-1t-1}, \ln p_{1t-1}, ..., \ln p_{nt-1}, \ln(e_{t-1} / P_{t-1}); g_t) \qquad [10]$$

where g_t is assumed to be an exogenous I(1) forcing variables. Without the need of identifying the short-run dynamics, Pesaran and Shin (2002) show that for (exact) identification of β it is necessary an order condition that requires $r^2 = (n-1)^2$ restrictions. Equivalently, this condition requires that there must be at least r independent a priori restrictions (including one normalization restriction) one each of the r cointegrating relationships. Thus, within the matrix of cointegration vectors $\beta^{'}$ we have:

$$\beta' = (\beta^{*'}, \theta) = [I_{n-1}, -B, -\theta] \qquad [11]$$

where I_{n-1} is the $n-1 \times n-1$ unitary matrix that serves for imposing the exact-identifying conditions in the coefficients of $n-1$ budget shares.

Though the rank identification conditions are testable since asymptotic distribution for the cointegration trace test of r is well established (Harbo et al., 1998, Pesaran et al., 2000), empirical time series estimations of demand systems are based on a (statistically) finite number of observations and may provide a poor guide for inference. This effect may be amplified when we do not use parsimonious (nested) models that incorporate restrictions in the long-run parameters. In fact, the interaction between dynamic adjustments and long-run specification may affect the rank test of identification as well as the size and power properties.

Given the setting with non-separability assumption, which we will use as the reference model, we specify a separable long run demand system assuming that the influence of public goods on consumer preferences through the θ_{ij} parameters is negligible. We argue that, in evaluating the identifying rank test, the separable specification should produce positive effects in large dynamic systems when the assumption is not rejected, while irrelevant effects should emerge when the model is rejected.

An alternative nested model imposes the homogeneity constraint in the earliest stage of specification, assuming that prices and nominal income are homogeneous of degree zero (Ng, 1995). After substitution in the *B* matrix, the cointegration vector is written as:

$$\beta' x_{t-1} = \beta'(w_{1t-1}, ..., w_{n-1t-1}, \ln p^{\circ}_{1t-1}, ..., \ln p^{\circ}_{n-1t-1}, \ln(e_{t-1} / P_{t-1}); g_t) \qquad [12]$$

where $\ln p^{\circ}_{i\,t-1} = \ln(\dfrac{p_{i\,t-1}}{p_{n\,t-1}})$

In this setting the reduction of stochastic trend sizes concerning price variables and the restriction of dynamic adjustments of the model allow predicting an increase of the power and a fall of the spurious rejection rate in small sample sizes of the cointegration test when the theoretical non-linear restrictions of homogeneity is not rejected by the data (Greenslade et al., 2002).

Below we examine the evidence on the identification rank procedure by simulating the cointegration test of the trace statistic for non-separable and (theoretically) unrestricted demand system by means of Monte Carlo procedures. Besides comparing the results of the cointegration rank and the power and size of the test, we assess the robustness of the cointegration rank tests of the reference model in simulated data (with optimal lags) against specifications with one added lag. We will return to the details when we will discuss the sensitivity of the models.

3. MONTE CARLO STUDY

3.1. Setup

The strategy adopted for evaluating identifying conditions of demand models by Montecarlo experiments consist in using a “tipical” framework of these models as the data generating process (DGP). This is deliberately chosen to be similar to the type of system estimated by Aristei and Pieroni (2008) with a sample size of 156 observations (1964q1-2002q4), incorporating three private categories: (a) Health, Education, Social Protection, Recreation and Culture; b) Services (including rents and rates); c) Food, Energy and other non-durables)[2]. The models are specified including a restricted intercept in the cointegrated equations as a proxy for the steady state equilibrium of the demand system and dynamic structures in line with the lags of the empirical works. In this setting, the exact identification of the empirical model requires two cointegrating vectors and, in structural form, each vector enters only the corresponding budget share equation. Further, to account for the heterogeneous effects of government spending on private expenditure allocation, we estimate

[2] The typical sample size of 156 time observations is similar to the sample used by Pesaran and Shin (2002) and Fanelli and Mazzocchi (2002) for separable long run demand systems.

two different non-separable long run demand system including aggregate government spending (G) and, separately, individual public consumption (GI) and collective government consumption (GC) components as weakly I(1) exogenous variables[3].

Our first aim is to conduct a series of Monte Carlo pair-wise comparisons of the cointegration rank test between the described non-separable reference model and nested specifications. Specifically, we generate predicted values for the left-hand side variable in the model using the regression results for the maintained "true" model. Then, we perturb each predicted observation with a zero-mean, normally distributed random number generator (with standard error set equal to the estimated root mean square error from the original regression). We repeat this perturbation process to generate Monte Carlo data sets[4]. The observation panels are assumed to be reasonable simulated observations of the non-separable dynamic demand systems, i.e. reference models, and are used to estimate the VAR models, to perform cointegrating rank tests and, after imposing the identification conditions of the equation [11], to assess the power and size property for the reference model and for separable and homogeneity-restricted nested specifications.

3.2. Results

Table 1 summarises the results of simulation experiments of the cointegrating rank for the model with optimal lags (p=2). The estimation procedure involves carrying out a succession of tests to determining the rank which consists of testing $H^*(0), H^*(1)...$ and determining the rank as the number of the first test that is not rejected. The simulated trace tests suggest the

[3] An issue with public expenditure data is that quarterly observations are available for the series of total public consumption only, while disaggregated information on the composition of public spending is available on annual basis. Quarterly data on individual and collective public consumption are obtained by disaggregating the relative annual series and comparing the results obtained by the Chow-Lin (1971), Fernandez (1981) Litterman (1983) and Santos Silva and Cardoso (2001) methods. Complete results and details on the disaggregation methods considered are available from the authors.
[4] We found that reducing the number of datasets starting from 1000 to 120 had no effect on any of the results. We use 250 iterations to obtain Monte Carlo data sets.

pervasive presence of two cointegrating relationships in different demand model specifications, so that we empirically do not reject the rank condition of exact-identification attributing an equilibrium relationship for each estimated equation of the system.

INSERT TABLE 1

To deepen the statistical properties of the trace test statistic, we calculate the probability of rejection of the null hypothesis of the cointegration rank, after imposing the exact order conditions of identification and comparing the reference model with the separable specification, using a number of alternative sets of critical values and adding an extra lag to the optimal dynamic.

INSERT TABLE 2

The results of this exercise are reported in Table 2 and show that the bias of the identification rank test does not increase by including G or its components (GI and GC) when the hypothesis of separability is not contained in the data. If we correctly specify the cointegration rank (2) and the order of VAR(2), we have an increase in the power of the test, always rejecting the assumption when this is false. The size properties of the trace tests indicate that the estimated actual size is significantly different from the nominal size of the test at the 5% level of significance. We reject this assumption for the true model 36-37% of the time when it is true in both the specifications for a nominal size of 5%. Thus, the performance of the test does not improve if we remove government spending to obtain separable specification, because of the presence of significant effects of government spending in private consumption, while either inefficiencies or under-identification are obtained when we assume that the cointegrating rank is larger (by one lag) than the true optimal value for unrestricted and separable models, respectively.

The upper part of Table 3 shows the separability test results, presenting both the standard asymptotic LR test and a small sample corrected version of the test to account for the over-rejection tendency of the test in small samples (Pudney, 1981)[5]. The results for the two conditional specifications are reported in Table 3. The values of the LR tests are higher than critical values and unambiguously indicate that consumer preferences are non-separable between publicly provided and privately purchased goods and services. We conclude that demand parameters can not be estimated reliably without regard to government spending and its potential effects on private consumption.

We have also argued that if the homogeneity imposition is contained in the data, so the identification of the conditional demand system starting from the "restricted specification" may improve the size property of this test. In the lower part of the Table 3, we report both the asymptotic and small sample corrected likelihood ratio tests for homogeneity restriction. In fact, Dufour and Khalaf (2002) find that in finite samples standard asymptotic results can be very misleading, since they are biased towards over-rejection when the number of equations and parameters of the models is large with respect to the sample size. As it is common in demand studies (see Ng, 1995), we find that the asymptotic test for the theoretical restrictions leads to a general rejection of the null hypothesis, with LR test statistics well above the corresponding 1% critical value. As it can be noted, after applying the small-sample correction, the test results changed with an evident improvement in the statistical significance of the homogeneity constraint for the model conditioned to total public consumption. In

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[5] The adjusted LR statistic is defined as: $LR^{*} = LR + nT\log\left[(nT - p_1)/(nT - p_0)\right]$, where *n* is the number of equations and p_0 and p_1 are the numbers of parameters of the restricted and unrestricted specifications, respectively. An analogous correction is also carried out for the critical values, which take the form $K = nT\log\left[1 + dF^{d}_{nT-p_1}/(nT - p_1)\right]$, where $d = p_1 - p_0$ and $F^{d}_{nT-p_1}$ is the critical value for the *F* distribution.

particular, the small-sample adjusted LR tests indicate that the theoretical restriction can be rejected at the 5% significance level for all the specifications considered, even if we are unable to reject homogeneity restrictions at the 1% level of significance.

INSERT TABLE 3

Figures 1.a and 1.b present the probabilities for a nominal size of 5% of the restricted and unrestricted non-separable models referring to the size of the test when the rank of matrix is equal to the true rank *(r=r*)*. We calculate the rejection probabilities for a sequence of significance levels ranging from .01 to .1 at steps of magnitude 0.01 to provide P-value (PV) plots (Davidson and MacKinnon, 1998). When the null hypothesis is not true we expect the plot to lie above the 45° line, while if the null is true the plot should be close to the 45° line. The standard errors of the estimated actual sizes are given by $\sqrt{N^{-1}\hat{\alpha}^{*}(1-\hat{\alpha}^{*})}$ where $\hat{\alpha}^{*}$ is the estimated actual size and N is the number of Monte Carlo replications. The results point to two major conclusions. Firstly, under the maintained hypothesis of homogeneity in the data, we have a systematic reduction of 10% of the size (rejection probability) of the cointegration trace test for the non-separable specification with homogeneity imposed with respect to the reference model. Secondly, the size properties of the cointegrating rank test are, however, over-rejected for the worth nominal size span in both of empirical specifications.

INSERT FIGURE 1

Finally, the simulated substitution or complementary effects of government spending on private consumption are reported in Table 4. We obtain robust elasticity results compared to the findings of Aristei and Pieroni (2008), showing that private consumption decisions are significantly affected by the public provision of goods and services and that the public sector has simultaneous *crowding out/in* effects on private consumption expenditures. We note that the private counterpart to individual government expenditure ("HER") is characterized by the highest substitutability value while substitution relationships with "Other non durables" is less pronounced. On the other hand, the elasticities of the "Services" category with respect to GI and GC show that an increase in the public provision of goods and services causes an increase in private spending. This result is in line with the findings of Karras (1994), Kuehlwein (1998) and Fiorito and Kollintzas (2004) concerning the possibility of having complementarity relationships between some components of public and private spending.

INSERT TABLE 4

4. CONCLUSIONS

This paper focused on identification issues of conditional cointegrated demand systems. The strategy of the analysis extends the Pesaran and Shin's (2002) identifying theory-based approach to account for the utility of government expenditure on the allocation of private consumption.

Monte Carlo experiments of this extended model are conducted by using the Johansen framework and modelling government spending components as exogenous non-stationary forcing variables. Then, under the exact-identification hypothesis, the size and power of the cointegration rank test for this benchmark model are compared with respect to separable demand systems and to homogeneity-restricted specifications.

The results document that the cointegrating rank tests do not reject the exact-identification when the optimal lags are included in each model. We find confirmation of the strong power of the cointegration rank test, though the empirical size is substantially higher than the nominal size. These statistical findings are not, however, affected by the inclusion of significant government effects.

Furthermore, we have shown significant statistical gains with systematic reduction of the empirical rejection rate of the test when the model with homogeneity restriction is imposed. In this case, the reduction of the risks of finding a spurious relationship among variables of the conditional demand system and of miss-specifying the results concerning the exact identification of the long run demand relationships makes this empirical specification preferable. Finally, the robustness of the estimations is furthers supported by the plausible values of the estimated elasticities of private expenditures with respect to public provision of goods and services.

REFERENCES

Aristei, D., Pieroni, L. (2008). Estimating the Role of Government expenditure in long-run consumption. *Scottish Journal of Political Economy* 55(2):143-166.

Chow, G., Lin, A.L. (1971). Best linear unbiased Interpolation, Distribution and Extrapolation of Time Series by Related Series. *The Review of Economics and Statistics* 53:372-375.

Davidson, R., MacKinnon, J.G. (2008). Graphical methods for investigating the size and power of test statistics. *The Manchester School* 66:1-26.

Dufour, J.M., Khalaf, L. (2002). Simulation based finite and large sample tests in multivariate regressions. *Journal of Econometrics* 111:303-322.

Deaton, A.S. (1981). Theoretical and empirical approaches to consumer demand under rationing, in A.S. Deaton, *Essays in the theory and measurement of consumer behaviour*. Cambridge, Cambridge University Press.

Deaton, A.S., Muellbauer, J. (1980). An almost ideal demand system. *American Economic Review* 70:312-326.

Fernandez, R.B. (1981). Methodological note on the estimation of time series. *Review of Economic and Statistics* 63:471-478.

Fiorito, R., Kollintzas, T. (2004). Public goods, merit goods, and the relation between private and government consumption. *European Economic Review* 48:1367-1398.

Greenslade, J.V., Hall S.G., Brian, H.S.G. (2002). On the identification of cointegrated systems in small samples: a modelling strategy with an application to UK wages and prices. *Journal of Economic Dynamics and Control* 26:1517-1537.

Harbo, I., Johansen S., Nielsen B., Rahbek, A. (1998). Asymptotic inference on cointegrating rank in partial systems. *Journal of Business & Economic Statistics* 16:388-399.

Kuehlwein, M. (1998). Evidence on the substitutability between government purchases and consumer spending within specific spending categories. *Economics Letters* 58:325-329.

Fanelli L., Mazzocchi, M. (2002). A cointegrated VECM demand system for meat in Italy, *Applied Economics* 34 (13):1593-1605.

Johansen, S. (1995). Identifying Restrictions of Linear Equations with Applications to Simultaneous Equations and Cointegration. *Journal of Econometrics* 69:111-132.

Karras, G. (1994). Government spending and private consumption: some international evidence. *Journal of Money, Credit and Banking* 26:9-22.

Fleissig, A.R., Rossana, R.J. (2003). Are Consumption and Government Expenditures Substitutes or Complements? Morishima Elasticity Estimates from the Fourier Flexible Form. *Economic Inquiry* 15:132-146

Lewbel, A., Ng, S. (2005). Demand Systems With Nonstationary Prices. *Review of Economics and Statistics* 87:479 - 494.

Litterman, R.B. (1983). A Random Walk, Markov Model for the Distribution of Time Series. *Journal of Business and Economic Statistics* 1:169-173.

Neary, J.P., Roberts, K.W.S. (1980). The theory of household behaviour under rationing. *American Economic Review* 13:25-42.

Ng, S. (1995). Testing for homogeneity in demand systems when the regressors are nonstationary. *Journal of Applied Econometrics* 10:147-163.

Pesaran, M.H., Shin Y. (2002). Long-Run Structural Modelling. *Econometrics Reviews* 21:49-87.

Pesaran, M.H., Shin Y., Smith, R.J. (2000). Structural analysis of vector error correction models with exogenous I(1) variables. *Journal of Econometrics* 97:293-343.

Pollak, R.A. (1969). Conditional Demand Functions and Consumption Theory. *The Quarterly Journal of Economics* 83:60-78.

Pollak, R.A. (1971). Conditional demand functions and the implication of separable utility. *Southern Economic Journal* 78:401-414.

Pudney, S.E. (1981). An empirical method of approximating the separable structure of consumer preferences. *Review of Economic Studies* 48:561-577.

Santos Silva, J.M.C., Cardoso, F.N. (2001). The Chow-Lin method using dynamic models. *Economic Modelling* 18:269-280.

Tridimas, G. (2002). The dependence of private consumer demand on public consumption expenditures. Theory and evidence. *Public Finance Review* 30:251-272.

Table 1 - Johansen's Cointegration Rank: Simulated Trace Statistic Test

a) Benchmark model: non-separable model conditional on GI and GC

No. of CE(s)		Benchmark Model	Nested models	
H_0	H_1		Separable	Homogeneity-restricted
$r = 0$	$r \geq 1$	210.657 (0.000)	169.759 (0.000)	136.204 (0.000)
$r \leq 1$	$r \geq 2$	112.343 (0.006)	78.049 (0.040)	87.735 (0.019)
$r \leq 2$	$r \geq 3$	54.602 (0.536)	42.741 (0.345)	48.653 (0.213)
$r \leq 3$	$r \geq 4$	27.705(0.858)	23.057(0.529)	18.798 (0.776)
$r \leq 4$	$r \geq 5$	11.019 (0.957)	7.842 (0.834)	6.064 (0.819)
$r \leq 5$	$r \geq 6$	5.219 (0.746)	2.480 (0.685)	-

b) Benchmark model: non-separable model conditional on G

No. of CE(s)		Benchmark Model	Nested models	
H_0	H_1		Separable	Homogeneity-restricted
$r = 0$	$r \geq 1$	210.004 (0.000)	173.393 (0.000)	137.793 (0.000)
$r \leq 1$	$r \geq 2$	111.187 (0.000)	77.375 (0.045)	86.818 (0.003)
$r \leq 2$	$r \geq 3$	55.298 (0.201)	43.876 (0.297)	46.941 (0.092)
$r \leq 3$	$r \geq 4$	27.573 (0.609)	23.198(0.520)	17.808 (0.611)
$r \leq 4$	$r \geq 5$	11.054 (0.828)	7.999 (0.822)	6.835 (0.562)
$r \leq 5$	$r \geq 6$	4.646 (0.607)	2.565 (0.669)	-

Notes: The cointegrating rank tests of the reference models are obtained by the Monte Carlo data set of the typical sample size (156 observations) and lags=2. It is worth remarking that the critical values used for testing the cointegration rank account for exogenous I(1) forcing values (Harbo et al. (1998); Pesaran et al. 2000).

Table 2 – Power and actual size of simulated trace statistic test

	Models (GI-GC)				*Models (G)*			
	Reference		Separable		Reference		Separable	
	p=2	p=3	p=2	p=3	p=2	p=3	p=2	p=3
r	***2***	***2***	***2***	***1***	***2***	***2***	***2***	***1***
Power and Size (1%)								
$r = 0$ *against* $r \geq 1$	100	100	100	-	100	100	100	-
$r \leq 1$ *against* $r \geq 2$	100	98.71	89.03	-	100	98.06	85.81	-
$r \leq 2$ *against* $r \geq 3$	16.13	24.52	16.03	-	16.77	26.45	14.84	-
Power and Size (5%)								
$r = 0$ *against* $r \geq 1$	100	100	100	-	100	100	100	-
$r \leq 1$ *against* $r \geq 2$	100	100	98.06	-	100	100	94.84	-
$r \leq 2$ *against* $r \geq 3$	37.42	54.84	38.06	-	36.77	50.97	37.42	-
Power and Size (10%)								
$r = 0$ *against* $r \geq 1$	100	100	100	-	100	100	100	-
$r \leq 1$ *against* $r \geq 2$	100	100	98.06	-	100	100	94.84	-
$r \leq 2$ *against* $r \geq 3$	55.48	70.97	54.19	-	54.84	71.61	52.26	-

Notes: *The identifying condition of a long run demand system (6) is evaluated by simulating cointegrating rank. Specifications (I) is the nonseparable reference model that include components of government expenditure as I(1) forcing variables, while Specification (II) is the separable counterpart. r is the simulated cointegrating rank, p is the length of the VAR used. Power of simulated data of the model is shown in the first two lines while the actual size is highlighted in the last line.*

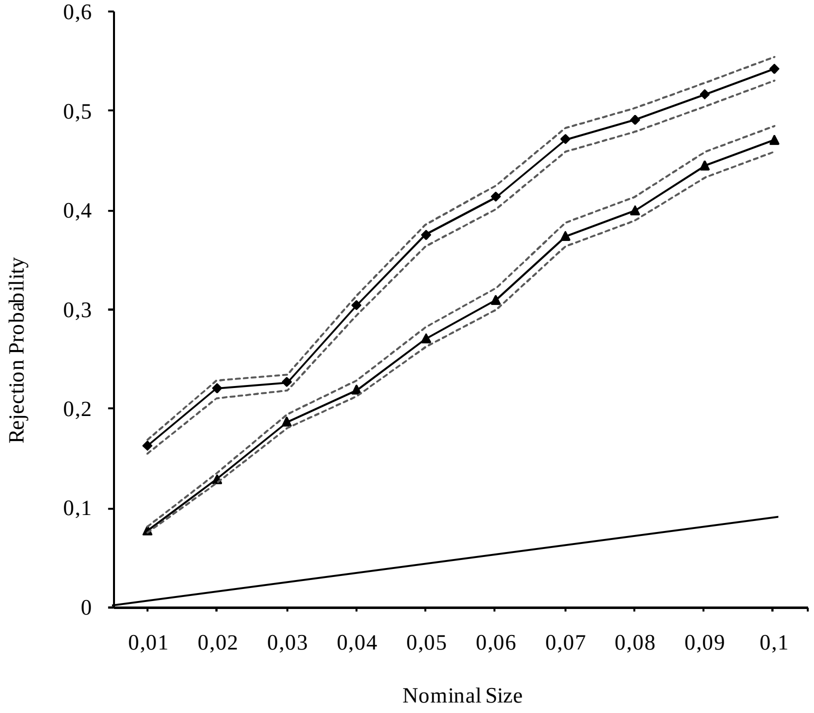
Table 3 – Tests for simulated nested models

	Standard asymptotic results		Small-sample correction		
Model Specification	LR statistic [χ^2]	p-value	Test statistic	5% critical value	1% critical value
1. H_0: Separability between private and public consumption					
Conditional specifications:					
a) G	20.234	0.000	16.07	4.92	9.64
b) GI+GC	23.623	0.000	19.43	4.95	9.69
2. H_0: Theoretical restriction of homogeneity					
Conditional specifications:					
a) G	12.536	0.002	10.46	7.91	13.67
b) GI+GC	13.488	0.001	11.40	7.93	13.71

Notes: small-sample corrected critical values are computed as in Pudney (1981)

Figure 1 – Size properties of the cointegration rank tests for reference model: PV Plots for Montecarlo simulation (with standard errors) (Non-separable long run demand system. True Rank: 2, H_0: Rank= 2)

a) Benchmark non-separable model conditional on GI and GC



b) Benchmark non-separable model conditional on G

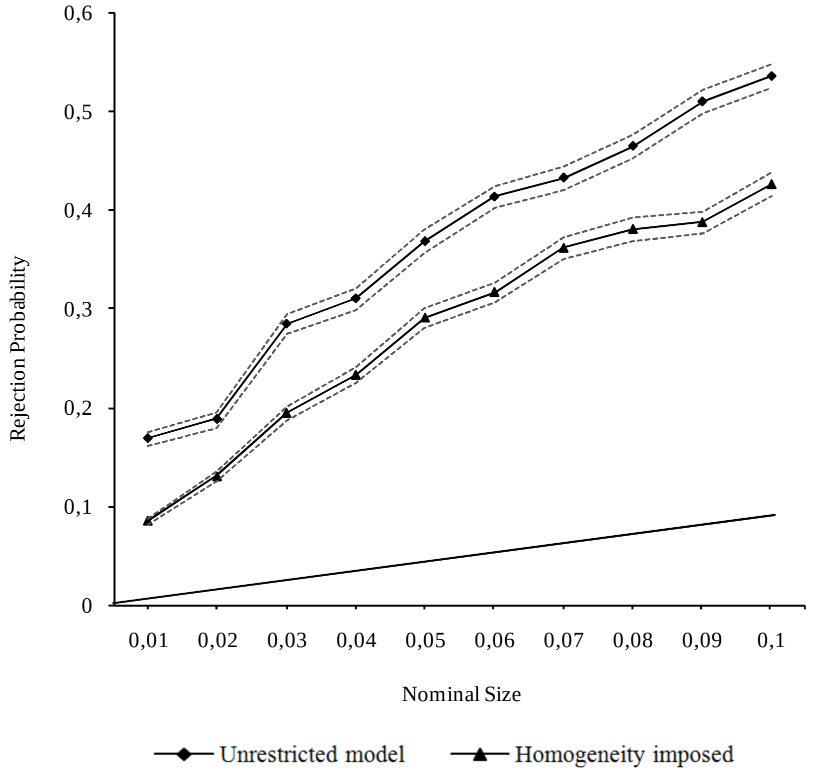


Table 4 – Estimated elasticities of the non-separable long run demand system

Models	*HER*		*Services*		*Other non-durables*	
	GI	GC	GI	GC	GI	GC
Unrestricted	-0.6358 (0.267)	-0.425 (0.182)	0.172 (0.155)	0.069 (0.108)	-0.027 (0.086)	-0.121 (0.059)
Homogeneity Restrictions	-0.601 (0.229)	-0.453 (0.163)	0.299 (0.107)	0.071 (0.145)	-0.151 (0.049)	-0.122 (0.104)
Hom. and symmetry Restrictions*	-0.685 (0.263)	-0.549 (0.182)	0.369 (0.163)	0.246 (0.109)	-0.176 (0.064)	-0.099 (0.057)

Notes: The estimated elasticities of the first two models are obtained by the simulations data, while the results of the model with homogeneity and symmetry imposed are those reported in Aristei and Pieroni (2008). Asymptotic standard errors are presented in round brackets.