

CIDE Lectures on Volatility Estimation and Modelling

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These lectures summarize the recent developments on realized volatility and related measures. After the introduction, we discuss the basic theory based on central limit theorems as well as on finite sample results. We then study the predictive ability of realized measures to forecast future volatility. The second lesson will also study the impact of jumps on volatility measures. The third lesson focuses on the impact of the presence of market microstructure effects on measuring and forecasting integrated volatility. We then use Edgeworth expansions to understand the finite sample properties of the central limit theorems in order to provide more accurate confidence intervals for the integrated variance. The last lesson is devoted to the analysis of estimating consistently covariances when observations are not synchronous, as well as to the estimation of continuous time processes when one uses realized measures.

1. Introduction and basic theory of realized volatility and related measures
2. (a) Modeling and forecasting realized measures
(b) Incorporating jumps
3. Realized volatility and market microstructure effects
4. Non-linear transforms, bootstrap and Edgeworth corrections for realized volatility
5. (a) An analysis of the bivariate case under non-synchronous observations
(b) Estimation of continuous time models based on realized measures

References

1- Introduction and basic theory of realized volatility and related measures:

Andersen, T.G., T. Bollerslev, F.X. Diebold and P. Labys (2001), "The Distribution of Exchange Rate Volatility," *Journal of the American Statistical Association*, 96, 42-55.

Andersen, T.G., T. Bollerslev, and D. Dobrev (2006), "No-Arbitrage Semi-Martingale Restrictions for Continuous-Time Volatility Models subject to Leverage Effects, Jumps and i.i.d. Noise: Theory and Testable Distributional Implications," *Journal of Econometrics*, forthcoming.

Barndorff-Nielsen, O., S.E. Graversen, J. Jacod, and N. Shephard (2006), "Limit Theorems for Bipower Variation in Financial Econometrics," *Econometric Theory*, forthcoming.

Barndorff-Nielsen, O. and N. Shephard (2002), "Econometric Analysis of Realized Volatility and its use in Estimating Stochastic Volatility Models," *Journal of the Royal Statistical Society, Series B*, 64, 253-280.

Jacod, J. and P. Protter (1998), "Asymptotic Error Distributions for the Euler Method for Stochastic Differential Equations," *Annals of Probability* 26, 267-307.

Meddahi, N. (2002), "A Theoretical Comparison Between Integrated and Realized Volatility," *Journal of Applied Econometrics*, 17, 479-508.

2a- Modeling and forecasting realized measures:

Andersen, T.G., T. Bollerslev, F.X. Diebold and P. Labys (2003), "Modeling and Forecasting Realized Volatility," *Econometrica*, 71, 579-625.

Andersen, T.G., T. Bollerslev, and N. Meddahi (2004), "Analytic Evaluation of Volatility Forecasts," *International Economic Review*, 45, 1079-1110.

Andersen, T.G., T. Bollerslev, and N. Meddahi (2005), "Correcting the Errors: Volatility Forecast Evaluation Using High-Frequency Data and Realized Volatilities," *Econometrica*, 73, 279-296.

Patton, A. (2006), "Volatility Forecast Comparison Using Imperfect Volatility Proxies," working paper, London School of Economics (<http://www.cireq.umontreal.ca/activites/060505/Papiers/patton.pdf>).

2b- Incorporating jumps:

Andersen, T.G., Bollerslev, T. and Diebold, F.X. (2005), "Roughing It Up: Including Jump Components in the Measurement, Modeling and Forecasting of Return Volatility," working paper, University of Pennsylvania (<http://www.ssc.upenn.edu/fdiebold/Volatility.html>).

Barndorff-Nielsen, O. and N. Shephard (2006), "Econometrics of Testing for Jumps in Financial Economics using Bipower Variation," *Journal of Financial Econometrics*, (2006), 4, 1-30.

Huang, X. and G. Tauchen, 2005, "The Relative Contribution of Jumps to Total Price Variance," *Journal of Financial Econometrics*, 3, 456-499.

Jacod, J. (2006), “Asymptotic Properties of Realized Power Variations and Related Functionals of Semimartingales,” working paper, Université de Pierre et Marie Curie (<http://www.crest.fr/seminaires/lfa/24MAi2006/Jacod.pdf>).

3- Realized volatility and market microstructure effects:

Aït-Sahalia, Y. and L. Mancini, “Out of Sample Forecasts of Quadratic Variations,” working paper, Princeton University (<http://www.crest.fr/microstructure/microstructure.htm>).

Aït-Sahalia, Y., P. A. Mykland, and L. Zhang (2005), “How often to Sample a Continuous-Time Process in the Presence of Market Microstructure Noise, *Review of Financial Studies*, 18, 351-416.

Andersen, T., T. Bollerslev, and X. Huang, “A Semiparametric Framework for Modelling and Forecasting Jumps and Volatility in Speculative Prices,” work in progress, Duke University (<http://www.cireq.umontreal.ca/activites/060423/Papiers/Huang.pdf>).

Andersen, T.G., T. Bollerslev, and N. Meddahi (2006), “Market Microstructure Noise and Realized Volatility Forecasting,” work in progress, Université de Montréal.

Bandi, F. M. and J. R. Russell (2005), “Microstructure Noise, Realized Variance, and Optimal Sampling,” working paper, University of Chicago (<http://gsbwww.uchicago.edu/fac/federico.bandit/research/BRa.pdf>).

Bandi, F. M. and J. R. Russell (2006), “Separating Microstructure Noise from Volatility,” *Journal of Financial Economics*, 79, 655-692

Barndorff-Nielsen, O., P.R. Hansen, A. Lunde, and N. Shephard (2006), “Designing Realised Kernels to Measure the Ex-post Variation of Equity Prices in the Presence of Noise,” working paper, Oxford University (<http://www.nuff.ox.ac.uk/users/shephard/realised.pdf>).

Hansen, P. R. and A. Lunde (2006), “Realized Variance and Market Microstructure Noise,” with discussion, *Journal of Business and Economic Statistics*, 24, 1272-18.

Huang, X. and G. Tauchen, 2005, “The Relative Contribution of Jumps to Total Price Variance,” *Journal of Financial Econometrics*, 3, 456-499.

Tauchen, G. and H. Zhou (2006), “Identifying Realized Jumps on Financial Markets,” working paper, Duke University (<http://www.cireq.umontreal.ca/activites/060423/Papiers/Tauchen.pdf>).

Zhang, L. (2006), “Efficient Estimation of Stochastic Volatility Using Noisy Observations: a Multi-Scale Approach,” *Bernoulli*, forthcoming.

Zhang, L., P. A. Mykland, and Y. Aït-Sahalia (2005), “A Tale of Two Time Scales: Determining Integrated Volatility with Noisy High-Frequency Data,” *Journal of the American Statistical Association*, 100, 1394-1411.

4- Non-linear transforms, bootstrap and Edgeworth corrections for realized volatility:

Gonçalves, S. and N. Meddahi (2005), “Bootstrapping Realized Volatility,” working paper, Université de Montréal (<http://www.mapageweb.umontreal.ca/meddahin/GM-July2005.pdf>).

Gonçalves, S. and N. Meddahi (2006), “Box-Cox Transforms for Realized Volatility,” working paper, Université de Montréal (<http://www.cireq.umontreal.ca/activites/060423/Papiers/Meddahi-I.pdf>).

Zhang, L., P. Mykland, and Y. Aït-Sahalia (2005), “Edgeworth Expansions for Realized Volatility and Related Estimators,” working paper, Princeton University (<http://www.princeton.edu/~yacine/edgeworth.pdf>).

5- An analysis of the bivariate case under non-synchronous observations

Griffin, J. and R. Oomen (2006), “Covariance Measurement in the Presence of Non-Synchronous Trading and Market Microstructure Noise,” working paper, University of Warwick (<http://www.cireq.umontreal.ca/activites/060423/Papiers/Oomen.pdf>).

Zhang, L. (2006), “Estimating Covariation : Epps Effect, Microstructure Noise,” working paper, University of Illinois at Chicago(<http://www.cireq.umontreal.ca/activites/060423/Papiers/Zhang.pdf>).

Hayashi, T., and N. Yoshida (2005), “On Covariance Estimation of Non-synchronously Observed Diffusion Processes,” *Bernoulli*, 11, 359-379.

Hayashi, T., and N. Yoshida (2006), “Estimating Correlations with Nonsynchronous Observations in Continuous Diffusion Models,” working paper, Columbia University (<http://www.cireq.umontreal.ca/activites/060423/Papiers/Hayashi.pdf>).

6- Estimation of continuous time models based on realized measures:

Barndorff-Nielsen, O. and N. Shephard (2002). Econometric analysis of realized volatility and its use in estimating stochastic volatility models. *Journal of the Royal Statistical Society, Series B*, 64, 253-280.

Bollerslev, T., M. Gibson, and H. Zhou (2004) “Dynamic Estimation of Volatility Risk Premia and Investor Risk Aversion from Option-Implied and Realized Volatilities,” working paper, Duke University (<http://www.cireq.umontreal.ca/activites/060423/Papiers/Zhou.pdf>).

Bollerslev, T. and H. Zhou (2002), “Estimating Stochastic Volatility Diffusion using Conditional Moments of Integrated Volatility,” *Journal of Econometrics*, 109, 33-65.

Corradi, V. and W. Distaso (2005), “Semiparametric Comparison of Stochastic Volatility Models using Realized Measures”, *Review of Economic Studies*, forthcoming.

Garcia, R., M.-A. Lewis, S. Pastorello, and E. Renault (2006), “Estimation of Objective and Risk-Neutral Distributions Based on Moments of Integrated Volatility,” working paper, Université de Montréal (<http://www.cireq.umontreal.ca/activites/060423/Papiers/Garcia.pdf>).

Feunou, B., R. Garcia, N. Meddahi, and R. Nguéfacq (2006), “Estimation of Continuous Time Models Based on Realized Measures: a Comparison of Methods,” work in progress, Université de Montréal.